27 $\hbar$ SQUID amplifier operating with high-Q resonant input load

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We have extended to ultracryogenic temperatures the complete noise characterization of a low-noise two-stage superconducting quantum interference device (SQUID) amplifier developed for resonant gravitational wave detectors. The additive current noise is evaluated from open input measurements. To evaluate the back action voltage noise, the SQUID is strongly coupled to a high-Q macroscopic electrical resonator operating at 11.7 kHz. From these measurements, we estimate a minimum noise temperature of 15 μK, corresponding to 27 times the quantum-limited noise temperature. Implications of this result for the sensitivity of resonant gravitational wave detectors are briefly discussed. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2168252]

The superconducting quantum interference device (SQUID) amplifier is a critical element for the sensitivity of present[1] and future[2] resonant gravitational wave detectors. The sensitivity of this type of detector to short gravitational wave bursts, expressed in terms of the effective temperature T_{eff} ($T_{\mathrm{eff}}=\Delta E_{\min}/k_B$, where $\Delta E_{\min}$ is the minimum detectable energy in the detector and k_B is the Boltzmann's constant), is limited by the noise temperature T_n of the SQUID amplifier used. The sensitivity limit is achieved when the resonant detector is lossless or when an optimal matching network between the bar, the sensing element of the detector, and the SQUID amplifier is used.[3]

As the amplifier noise temperature (or its equivalent energy resolution $\varepsilon=k_BT_n/\omega_0$, where $\omega_0/2\pi$ is the operation frequency) depends not only on the additive noise, but also on the back action noise, it is necessary to be able to evaluate both the spectral density S_{ii} of the additive current noise and the spectral density S_{vv} of the back action voltage noise [see Fig. 1(a)] in order to develop a SQUID amplifier for resonant detectors. In addition, the detector SQUID amplifier must be able to operate in a stable way with a high-Q macroscopic input load without seriously compromising the noise performance. The additive noise S_{ii} is obtained from simple open input noise measurements. From noise measurements in which the SQUID amplifier is strongly coupled to high-Q electrical resonators,[4] one can evaluate the back action noise S_{vv} at the resonator resonance frequency ω_0. The SQUID amplifier energy resolution is given by $\varepsilon=k_BT_n/\omega_0 \cong [S_{vv}S_{ii}]^{1/2}/2\omega_0$.

This letter describes the noise measurements performed at ultracryogenic temperatures on a low-noise two-stage SQUID, the type used on the AURIGA detector[5] which is currently operating at 4.5 K. These measurements have permitted evaluation of the noise temperature of the SQUID amplifier coupled to a high-Q macroscopic electrical resonator that simulates the input load constituted by the resonant detector.

In the two-stage SQUID amplifier, based on a commercial SQUID chip,[6] the signal of the first SQUID, the sensor SQUID, is amplified by the second SQUID, the amplifier SQUID. After further amplification and filtering by room-temperature electronics, the signal is fed back to the sensor SQUID [see Fig. 1(b)].[7] A cold damping network[8] between the input coil and the feedback line permits, without adding noise, to avoid negative Q instabilities when the two-stage SQUID is strongly coupled to a high-Q resonator. The input coil inductance of the sensor SQUID and its mutual inductance with the SQUID loop were measured to be $L_i = 1.615\pm0.010$ μH and $M_i=10.66\pm0.01$ nH. The resonator is composed of a low loss capacitance made by seven Teflon

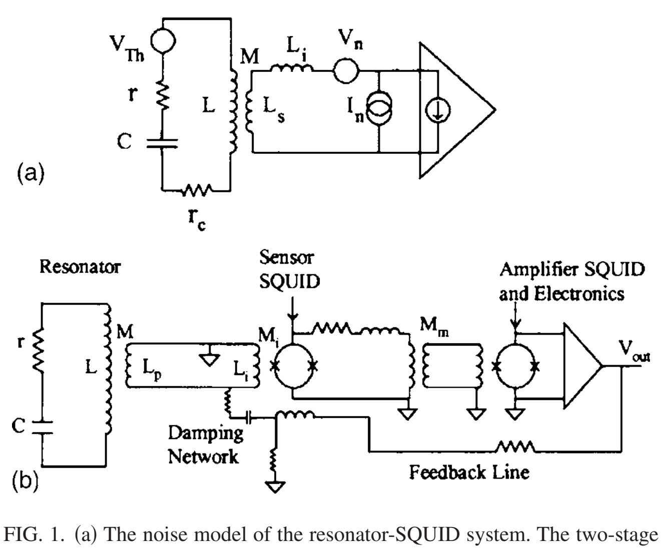

FIG. 1. (a) The noise model of the resonator-SQUID system. The two-stage SQUID is modeled by an ideal current amplifier with the noise sources V_n (back action noise) and I_n (additive noise). (b) Schematic circuit diagram of the resonator-SQUID system. The flux gain between the first and second SQUID is adjusted by means of the superconducting transformer M_m.

[a)]Electronic mail: falferi@science.unitn.it commercial capacitors in parallel for a total capacitance $C=32.9$ nF and a NbTi superconducting coil of inductance $L=10.2$ mH measured in its superconducting housing. The pickup coil is made by winding 20 turns of NbTi wire on the coil with an estimated inductance $L_p \cong 3.4$ μH. The mutual inductance $M=152$ μH between the resonator coil and the pickup coil has a coupling constant $k\cong 0.82$. A NbTi single turn coil is also wound on the resonator coil in order to excite the resonator with an external signal and then let it freely decay to measure its quality factor. The two-stage SQUID and the resonator are housed in a superconducting box which contains ^{4}He exchange gas and is attached to the mixing chamber of a dilution refrigerator. Other experimental details on the realization of the low loss resonator and its housing are reported in a previous paper.[9]

The expected resonance frequency, considering also the coupling to the SQUID, is about 11 kHz. We have chosen to operate at this frequency instead of around 1 kHz, the typical operation frequency of the present resonant gravitational wave detectors, for two reasons: First, to enhance the SQUID back action noise, which increases with the square of the frequency, over the resonator thermal noise; second, at 11 kHz the seismic and ambient vibrational noise are negligible in our experimental conditions. From our experience, the results can be extrapolated at lower frequencies as long as the $1/f$ contributions of S_{ii} and S_{vv}/ω_0^2 are negligible. In addition, one has to consider that the resonant detectors of the next generation[2] will operate with a larger bandwidth, 2–5 kHz, where the $1/f$ noise contribution of this SQUID amplifier is negligible.

Two ultracryogenic runs were performed to evaluate the noise temperature of the SQUID amplifier: One to measure S_{vv} at the resonator frequency and one to measure S_{ii} and the resonator intrinsic quality factor Q_i that is the quality factor measured without any effect due to the dynamic input impedance of the SQUID.

In the run for the measurement of S_{vv}, the SQUID input coil L_i is connected to the pickup L_p and the SQUID amplifier is strongly coupled to the resonator. This coupling changes the apparent quality factor of the resonator but, thanks to the cold damping network, the system is stable and the SQUID output noise shows a Lorentzian peak with quality factor $Q \ll Q_i$. The apparent quality factor, Q, is adjusted by properly choosing the values of the elements of the cold damping network. The noise model shown in Fig. 1(a) has been well established in similar systems at higher temperatures. In addition to the SQUID noise, the model considers only the resonator thermal noise V_{th} with spectral density $4k_BTr$ due to the resonator intrinsic losses represented by the resistance, r. A noise-free resistor, r_c, is included in the model to take into account the effect of the real part of the SQUID dynamic input impedance and the effect of the cold damping network. From this noise model, the amplitude $a(\omega_0)$, expressed in Φ_0^2/Hz, of the Lorentzian peak $a(\omega_0)/[(1-(\omega_0/\omega)^2)^2+(\omega_0/\omega Q)^2]$ in the SQUID flux noise spectrum is given by

$$a(\omega_0)=\left(\frac{MM_i}{L_t\Phi_0}\right)^2\left(\frac{4k_BT}{\omega_0 L_r Q_i}+\left(\frac{M}{L_t\omega_0 L_r}\right)^2 S_{vv}(\omega_0)\right), \qquad (1)$$

where M is the mutual inductance between the resonator coil and the pick-up coil, $L_t=L_i+L_p$, $L_r=L-M^2/L_t$ is the resonator coil inductance reduced by the coupling to the SQUID, $\omega_0=(L_rC)^{-1/2}$ is the resonance angular frequency, $Q_i=\omega_0L_r/r$ is the intrinsic quality factor, and $Q=\omega_0L_r/(r/r_c)$ is the apparent quality factor. From the value of $a(\omega_0)$ obtained with a fit of the averaged noise spectrum, $S_{vv}(\omega_0)$ can be evaluated if L_r, M/L_t, and Q_i are known. The values $(M/L_t)^2=917\pm 5$ and $L_r=(5.58\pm 0.01)$ mH were obtained from a calibration measurement at 4.2 K in which the resonator capacitance was replaced by a resistance of known value. L_r can be obtained from the low-pass cut-off frequency of the output SQUID noise spectrum; from its low-frequency amplitude, one can derive M/L_t. The Q_i measurements are described below.

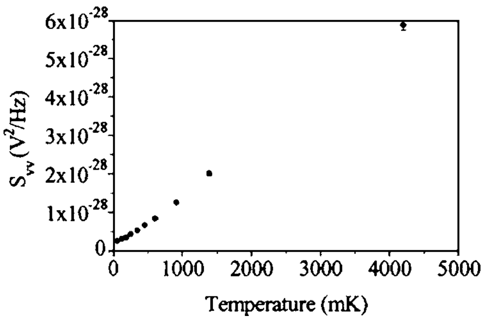

FIG. 2. SQUID noise spectral density S_{vv} at 11655 Hz as a function of temperature.

This measurement of S_{vv} has been carried on under the severe hypothesis that the resonator thermal noise is the only noise present, aside from that of the SQUID, down to the operation temperature of 55 mK. This condition has been recently demonstrated[9] with the same experimental apparatus but with a weaker coupling between SQUID and resonator in order to permit the measurement of the resonator thermal noise without adding noise or changing the intrinsic quality factor.

In the back action noise measurements, we used a bias current of the sensor SQUID that is higher than that which would minimize the noise of the SQUID operated open input, in order to overcome the system instabilities that are probably due to high-frequency spurious resonances in the resonator coil. The resulting resonance frequency was 11 655 Hz. In Fig. 2, the values of the back action noise S_{vv} at this resonance frequency are reported as a function of the temperature.

In the run for the measurement of S_{ii}, the sensor SQUID operates with open input coil, and the operation temperatures are the same as the back action noise measurements. Also the bias current of the sensor SQUID is kept the same as that in the back action noise measurements. We have chosen to operate in this way in order to evaluate the SQUID energy resolution with operating conditions as close as possible to those of the gravitational wave detector, that is with a high-Q resonant input load. Figure 3 shows the temperature behavior of the additive noise S_{ii} in the range of 10–11 kHz. The best value obtained at $T=55$ mK, $S_{ii}=6.8\times 10^{-27}$ A^2/Hz, corresponds to 4.26×10^{-7} $\Phi_0/\sqrt{\text{Hz}}$ at the SQUID loop. For a comparison, the best noise obtained at the same temperature with the optimal bias current is 3.36×10^{-7} $\Phi_0/\sqrt{\text{Hz}}$.

In the same ultracryogenic run, we also performed measurements of the intrinsic quality factor with a weakly coupled pickup which is part of the matching superconducting transformer between the two SQUIDs. This pickup, not shown in Fig. 1 but described in detail in Ref. 10, is in series

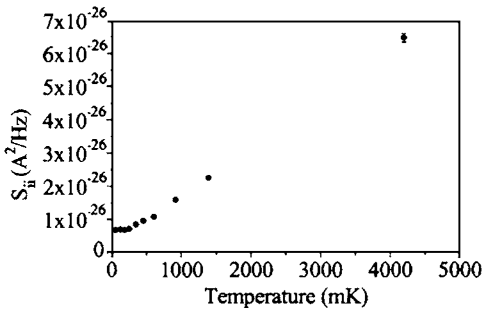

FIG. 3. SQUID noise spectral density S_{ii} in the frequency range of 10–11 kHz as a function of temperature.

with the secondary of M_m and the input coil of the amplifier SQUID and permits the amplifier SQUID (the sensor SQUID is turned off) to measure the resonator Q_i without perturbing it with its dynamic input impedance. As, in these measurements, the sensor SQUID is not coupled to the resonator, the inductance of its coil is not reduced. To operate the resonator and evaluate its Q_i at the same frequency of the back action noise measurements, we have inserted a high-purity niobium cylinder (h=70 mm, Φ=12 mm) into the coil support in order to reduce the coil inductance without adding significant losses. In this way, the obtained resonance frequency of 10 370 Hz was close enough to that used for the back action noise measurements. As reported in a previous work,[9] the measured Q_i depends significantly on the temperature below 1 K. The values range from 590 000 at 4.2 K to 219 000 at 55 mK. The reason for this unexpected behavior is still not understood.

Figure 4 shows the energy resolution ε of the two-stage SQUID amplifier as a function of the temperature derived from the values of S_{ii} and S_{vv} reported in Figs. 2 and 3. Of course, ε also presents a linear behavior at higher temperatures and saturation under 250 mK. The best measured energy resolution is 27 $\hbar$ at 55 mK which improves the previous best result[10] obtained at 1.33 K by about a factor of 5.

A comparison with the SQUID noise theory[11] is difficult and of limited significance, because in both the additive and back action noise measurements the bias current used is that which stabilizes the locking when the SQUID is strongly coupled to the resonator and not that which minimizes the noise. However, one can note that, regarding the data which scale with the temperature ($T \geqslant 0.6$ K), both S_{ii} and S_{vv} have values in excess with respect to the theory by about a factor of 4–5 and similar to those previously obtained.[10] Regarding the saturation of S_{vv} and S_{ii} below about 250 mK, the data do not permit us to distinguish between a saturation due to the hot electron effect[12] or other mechanisms which limit the thermalization of the SQUID amplifier.

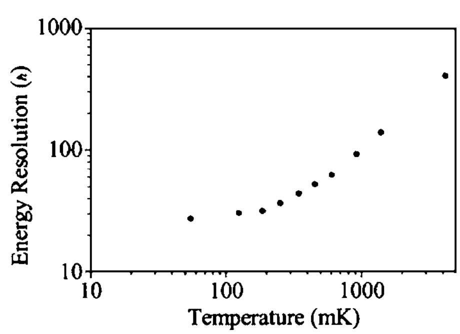

FIG. 4. SQUID energy resolution in number of quanta at 11 654 Hz as a function of temperature.

In this type of SQUID, the $1/f$ component of the additive noise does not depend significantly on the temperature below 1 K. At 55 mK and at 1 kHz, the $1/f$ contribution is about 50% of the white noise S_{ii} at 10 kHz. From previous measurements,[13,10] it is reasonable to expect a similar behavior also for S_{vv}. In this case, the sensitivity—expressed as the effective temperature T_{eff}—expected for the AURIGA detector operating at 100 mK and other operating parameters being equal, would improve from the present 300 μK to 10 μK. This sensitivity is still worse than the Giffard's limit[3] given by $T_{\text{eff}}=T_n \approx 2$ μK which could be achieved with the optimal noise matching between bar and SQUID amplifier.

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[1] For up to date results, see ⟨http://gravity.phys.lsu.edu⟩; ⟨http://www.auriga.lnl.infn.it/⟩; ⟨http://www.roma1.infn.it/rog/explorer/⟩; ⟨http://www.roma1.infn.it/rog/nautilus/⟩.
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