IGEC toolbox for coincidence search

L. Baggio†‡, M. Cerdonio§, I.S. Heng‖, A. Ortolan¶,
G.A. Prodi†, E. Rocco†, G. Vedovato¶ and S. Vitale†

† Univ. of Trento and INFN, via Sommarive, 14, 38050, Povo, TN, Italy
§ Univ. of Padova and INFN, via Marzolo, 8, 35131, Padova, PD, Italy
‖ Department of Physics and astronomy, Louisiana State University, 70803,
Baton Rouge, Louisiana, USA
¶ INFN Legnaro National Labs, 4, via Romea, 35020, Legnaro, PD, Italy

Abstract. The standard IGEC approach to detection of gravitational waves
with many detectors is simple time coincidence search. We discuss the problems of
false alarm and false dismissal assessment, both in the case of stationary and non-
stationary noise. The significance of any cumulative excess of found coincidences
over the background is determined by maximum likelihood methods.

1. Introduction

Gravitational waves (GW) resonant detectors operating so far are narrow band
detectors, and their typical bandwidths are no more than a few Hz with a central
frequency of about 900 Hz [1]. Therefore, they are insensitive to any structure present
in the waveform of the impinging GW, they just sample the amplitude of the Fourier
component of the signal at a few discrete frequencies. The impulse response template
is routinely used as a first approximation to describe all burst classes whose duration
is of the order of one millisecond.

The search for bursts is performed by threshold crossing (the signal overcome
a certain amplitude threshold)[2, 3, 4] or maximum-hold (the signal is locally at
maximum amplitude)[5] methods. This search usually allows very low ($\sim 3 \div 5$) signal-
to-noise ratios (SNRs). Estimating false alarms by relying on the nominal probability
of local fluctuation of a Gaussian stochastic process is often not viable, because outliers
(for instance, due to environmental transient noise) are unavoidable, and they mimic
the effect of GW signals. The puzzle can be solved when two or more detectors are
working at comparable sensitivity. Given that the delay between successive triggers is
much longer than the time uncertainty on the arrival time of the burst, the chances of
an accidental coincidence are proportionally low, the more the detectors the better.

A fundamental step before performing a coincidence search is to uniform the
sensitivitiy of the detectors by restricting their periods of operation such that they
overlap and that thet are all working with minimum detectable amplitudes below a
common search threshold. We assume that this step has already been done (further
details can be found in [6]). Also, we will not discuss here the synthesis of the
information on the GW burst provided by different detectors once a coincidence is
found.

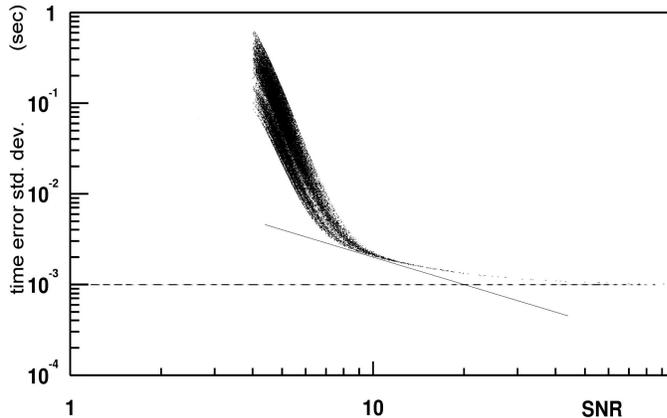‡ To whom correspondence should be addressed (lucio.baggio@lnl.infn.it)

Figure 1. Standard deviation of the time error vs. SNR for triggers exchanged within IGEC by the AURIGA collaboration (courtesy of AURIGA group). The continuous line is the asymptotic scaling of the standard deviation as $1/SNR$ expected at high SNR, the dashed horizontal line represents the lower bound due to present systematic calibration errors. Notice that 88% of the events have SNR between 5 and 6. Spreading at low SNR is due to variations of the Wiener filter bandwidth.

In this paper we report on coincidence search analysis as it is carried on within the International Gravitational Event Collaboration (IGEC)[1, 6]. First we give an operative definition of coincidence, and explain how the choice of a time window is linked to estimates of timing uncertainty through the requirement of a minimum detection efficiency. We shall describe how the accidental coincidences are estimated in section 3. The maximum likelihood criterion will eventually guide us to the assessment of detection confidence (section 4). The procedure we propose avoids unphysical (i.e. negative) values for the GW rate and it estimates the correct non-Bayesian confidence intervals for any number of found and expected coincidences, overcoming the criticisms to previous analyses of upper limits for GW rate (recently raised for instance by [7]).

2. Triggers, coincidences and time windows

The basic ingredients of IGEC GW search recipe are the self-consistent "*event files*", which are temporally ordered records of every *event* (or *trigger*) selected by each group of the collaboration as candidate gw signals [1]. It is understood that, before the event search, the data are processed by a Wiener filter matched to the impulse response of the system, what we call a $\delta - like$ GW. The parameters that fully describe such signal are just the estimated time of arrival (ETA) and amplitude together with their error statistics, and —when available— a test statistic to assess compliance of the signal with the template. Bundled in the same exchanged files, there are asynchronous information about amplitude precision and accuracy, and the values of the amplitude selection threshold.

The sequence of ETA in an event list can be described by a (possibly non-homogeneous) Poisson point series, which requires the triggers to be *rare* and *independent*. All search algorithms associate to every found trigger a dead time of the order of the reciprocal of the Wiener filter bandwidth, yet, as it is much shorter than the average delay between events, it does not affect significantly ETA statistics.

A coincidence is defined as a M-tuple of triggers, one for each detector, with ETA such that there is a common overlap between their *time windows*. More precisely, if $\mathbf{t}^{(m)}$ is the set of ETAs from detector labelled m, the following relation defines the set of all coincidences:

$$\left\{ \boldsymbol{c}_n = (t_1, t_2, \ldots, t_M) \,|\, t_m \in \mathbf{t}^{(m)} \,\&\, \forall h, k : |t_h - t_k + \Delta t_{hk}| < (\Delta t_h + \Delta t_k) \right\}_{n=1,2,\ldots} \quad (1)$$

where Δt_m is half the length of the time window associated with the event occurring at time t_m in the mth detector, and Δt_{hk} corrects for time delay in signal propagation at light speed. Δt_m is a function of the probability density of the timing error once a minimum confidence level (or a maximum false dismissal probability) has been fixed. This function is non-analytical and depends on the implementation of trigger selection and on the SNR of the signal, and it has to be determined by Monte Carlo methods. Figure 1 shows a typical scaling of the time error standard deviation σ with respect to SNR. The exact relation between σ_m and Δt_m depends on the robustness of the model for the statistics of the noise. The Gaussian model would give $\Delta t_m = 1.96\sigma_m$ (for a confidence level $c = 95\%$) but in general this model does not apply. Using the very general Bienaymè-Tchebicheff inequality, $c \leq 95\%$ when $\Delta t_m = \sigma_m/\sqrt{1-c} \approx 4.5\sigma_m$. This relation can be refined by including moments of higher order [8].

If the source direction is not available, its determination and consistency tests on ETA sequence are deferred to a later analysis, and the relation $|t_h - t_k + \Delta t_{hk}| < (\Delta t_h + \Delta t_k)$ in (1) is substituted by $|t_h - t_k| < \left(\Delta t_h + \Delta t_k + \Delta t_{hk}^{MAX}\right)$, where Δt_{hk}^{MAX} is the light travel time between the labelled pair of detectors.

3. False alarms estimates

A simple analytic formula for the expected background rate of accidental coincidences with the definition (1) is viable only by fixing for each detector the value of the standard deviation of the timing noise.

If the background triggers can be modelled as a homogeneous Poisson point process, and being $\lambda^{(k)}$ and Δt_k respectively the background rate and time window of the kth (over M) detector, the expected false alarm rate λ_b is given by†

$$\lambda_b = C_M(\Delta t_1, \ldots, \Delta t_M) \times \prod_{k=1}^{M} \lambda^{(k)} \quad (2)$$

where

$$C_M(\Delta t_1, \ldots, \Delta t_M) \equiv \sum_{k=1}^{M} \prod_{h \neq k} 2\Delta t_h \quad (3)$$

† To figure out how this formula can be derived, the reader should refer to figure 2, where the cases $M = 2$ and $M = 3$ are depicted. The measure of the fiducial volume is the product of its transverse section by its length. The first factor is given by the measure of the $M - 1$ dimensional surface obtained by projecting the M-dim error box in the plane transverse to the bisector. This means to multiply $1/\sqrt{M}$ to the measure $\prod_{h \neq k} 2\Delta t_h$ of each hyper-face, and summing over all directions $k = 1, ..., M$. The second factor is equal to the common observation time of the M detectors divided by the same quantity $1/\sqrt{M}$, which then cancels out in the product. In order to get the number of accidental coincidences, the measure of the fiducial volume have to be multiplied by the density of the lattice nodes representing each possible coincidence, which is given by $\prod_{k=1}^{M} \lambda^{(k)}$. Dividing by the common observation time gives at last (2).
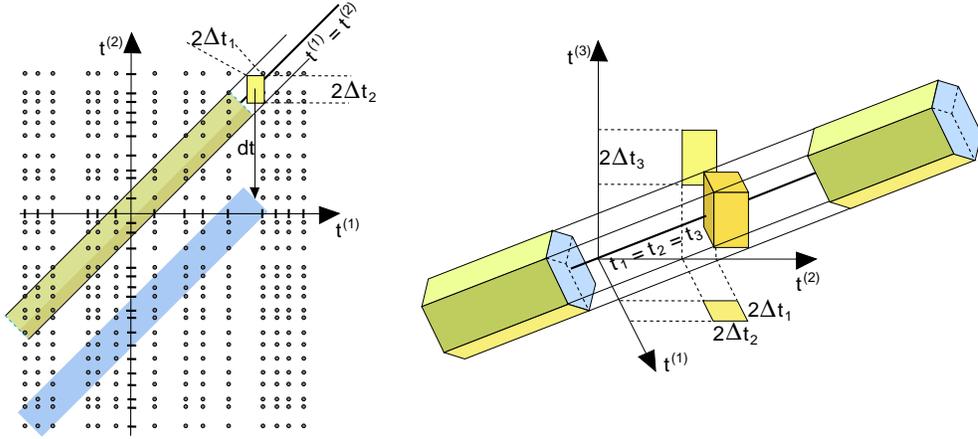
Figure 2. (*left*) The set of all possible pairings between two triggers in a two-detectors configuration can be represented with a lattice in 2-dim plane, obtained by tensor product of the original single detector event lists. In the simplified case when every trigger of one detector has the same time window, coincidence search can be seen as a particular selection of events in a stripe (*fiducial volume*) along the bisector, which corresponds to synchronous detector timescales. A time-shifted search means to count trigger pairs that fall inside a new translated stripe. (*right*) In a similar way, in a three-detectors configuration, the geometrical representation of the fiducial volume for coincidence search is a tube obtained developing the three dimensional error box along the bisector.

Equation (2) holds also as *instantaneous* background rate predictor when λ_b is a function of time. The average rate $\bar{\lambda}_b$ in the total observation time T_{obs} is given by

$$\lambda_b(t) = C_M(t) \times \prod_{k=1}^{M} \lambda^{(k)}(t) \quad \Leftrightarrow \quad \bar{\lambda}_b = \frac{1}{T_{obs}} \int dt\, C_M(t) \prod_{k=1}^{M} \lambda^{(k)}(t) \qquad (4)$$

In practical use of this result, the observation time is divided in smaller time intervals where the background process can be considered stationary, but long enough to estimate the statistics of the detector noise parameters. We remark that if the statistics of false alarms in each time interval is Poisson, then the total number of false alarms is again a Poisson random variable (RV), because it results from linear operations (like integrals). An issue still undecided with (4) is the implementation of a robust unbiased estimator of $\lambda^{(k)}(t)$ capable of dealing with low event rates.

A different well known numerical method to estimate false alarms probability is time-shifted search strategy. It consists in generating independent detector configurations by adding small delays dt to the ETA of all but one detector trigger lists. These configurations are supposed to be independent when the delay is applied in steps longer than the maximum total time coincidence window: ergodicity of the system is then required. In fact, referring to figure 2, the counts of coincidences inside each translated stripe are in principle different RV, but we use them as independent instantiations of the counts inside the synchronous stripe. This is correct only if detector performances at shifted times are *on average* the same as at zero delay. More precisely, the longest time shift should be shorter than the timescale of the structures of the cross-correlation between $\lambda^{(h)}(t)$ and $\lambda^{(k)}(t + dt)$ as a function of dt. The variations of the measured observation time T_{obs} itself are less critical, as they can are

easily taken into account by proper normalization of the results.

The time shift analysis is computationally heavier than simply using (4), but it allows to estimate the false alarm rate even when the value of the coincidence window varies from trigger to trigger, dealing with variations of SNR and of the Wiener filter bandwidth (see figure 1). Moreover it naturally keeps care of the non-homogeneous character of the trigger rate. Finally, we remark that, by repeated independent time shifts, the Poisson statistics of the coincidence counts can be directly tested, instead of being blindly assumed. For these reasons, the number of accidental coincidences is presently estimated within IGEC by time shift analysis, through the analytical approach is perhaps worth a further thought.

Note that either method (analytic or numerical) is biased by the assumption that the event list contains no real signal, yet since the latter are supposed to be much more rare, this effect is negligible.

4. Computation of average GW burst rate

At last, when the number of predicted and found coincidences have been determined, a decision has to be made about the presence among the latter of gw signals. In the our non-Bayesian approach, a straightforward result in terms of confidence intervals is obtained by maximum likelihood methods.

Given the mean number of expected background coincidences $\bar{\mathbf{N}}_b \equiv E\{\mathbf{N}_b\}$ over a time T_{obs}, let Λ be the unknown mean rate of triggers produced by random (and rare) sources, which is natural to model with a Poisson point process. Even if it happens to be non-homogeneus, the number of coincidences N_c counted in a typical experiment is a statistical sample of a Poisson RV $\mathbf{N}_c$ with mean $\bar{\mathbf{N}}_c = \bar{\mathbf{N}}_b + T_{obs}\Lambda$. Let it be $N_\Lambda \equiv T_{obs} \cdot \Lambda$. The probability density function of $\mathbf{N}_c$ computed at the observed value is

$$P_{N_c}(\bar{\mathbf{N}}_b + N_\Lambda) \equiv \frac{1}{N_c!}\left(\bar{\mathbf{N}}_b + N_\Lambda\right)^{N_c} e^{-\left(\bar{\mathbf{N}}_b + N_\Lambda\right)} \tag{5}$$

and the corresponding likelihood function is

$$\ell(N_\Lambda; N_c, \bar{\mathbf{N}}_b) \equiv P_{N_c}(N_\Lambda + \bar{\mathbf{N}}_b) \tag{6}$$

with the obvious bound $N_\Lambda \geq 0$. The most likely value of N_Λ supported by the observations is obtained when ℓ is maximum, and for any *confidence level* (CL) $c \in [0,1]$, its associated *confidence interval* $[N_{\inf}, N_{\sup}]$ is defined by $\ell(N_{\inf}) = \ell(N_{\sup})$ and $c = \left[\int_0^\infty \ell(N)dN\right]^{-1}\int_{N_{\inf}}^{N_{\sup}}\ell(N)dN$. We shall say that the number of found coincidences N_c "*agrees at confidence level c with a rate Λ between $N_{\inf}/T_{obs}$ and $N_{\sup}/T_{obs}$*". In the case $N_{\inf} = 0$ the null hypothesis is not ruled out by the experiment, and we shall say "*$N_{\sup}/T_{obs}$ is the upper limit* at confidence level *c*". Our non-Bayesian interpretation of this statement is that the actual outcome is $c/(1-c)$ times more likely to be observed over many repetitions of the experiment if the value of N_Λ is inside the interval $[N_{\inf}, N_{\sup}]$, than if it were outside.

Figure 3 shows an example of confidence intervals set as a function of the number of found coincidences for the case $\bar{\mathbf{N}}_b = 7.0$. Assuming that 1000 time shifts were performed (a typical figure for $M = 2$ in the current IGEC analysis), the standard deviation of the estimate of $\bar{\mathbf{N}}_b$ is 0.083, which propagates to the confidence intervals like depicted in Figure 3. Up to $N_c = 12$ no positive detection claim is allowed at 95% CL, but only upper limits.
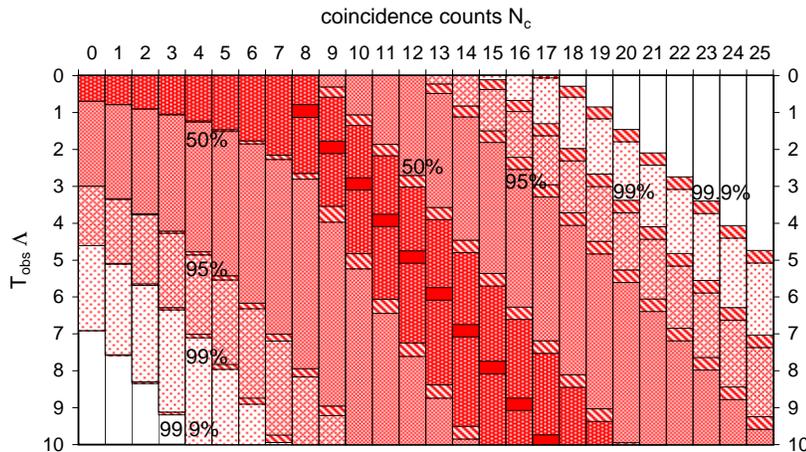
Figure 3. Example of confidence level (CL) contour plot for a background $\bar{N}_b = 7.0$. For each possible integer value of N_c in the abscissa, the corresponding confidence interval can be determined by finding along the vertical real axis the intercepts with a pair of contours, enclosing the desired CL. In this example, the 50%, 95%, 99% and 99.9% CL contours are plotted, along with the most likely value (*darkest color*). The dashed regions show the amount of fluctuation of the contours when the estimate of $\bar{N}_b$ falls within 6.83 and 7.17, corresponding to four times the standard deviation after 1000 independent shifts. Notice that when $N_c < \bar{N}_b$ the likelihood function becomes single-tailed, and the most likely value is zero.

This procedure can be retraced with a different density function in case of known deviations of the data from the Poisson statistics.

5. Conclusions

Within IGEC, the presently used coincidence search method varies the time window aperture in order to guarantee a specific maximum false dismissal. The corresponding false alarm statistics is then empirically investigated by assuming the ergodic approximation and taking time-shifted configurations of the observatory as independent instantiations of the observations at zero time delay. Finally, by feeding the standard maximum likelihood with the estimated false alarm rates, the significance of the estimated GW average burst rate can be assessed.

References

[1] Allen B Z *et al* 2000 *Phys. Rev. Lett.* **85** 5046; see also http://igec.lnl.infn.it/
[2] Mauceli E *et al* 1996 *Phys. Rev.* D **54** 1264; Heng I S 2001 Private communication.
[3] Astone P *et al* 1999 *Astrop. Phys.* **10** 83
[4] Heng I S, Tobar M E and Blair D G 1999 *Class. Quantum Grav.* **16** 3439
[5] Ortolan A et al 2002 *Class. Quantum Grav.* **19** and references therein.
[6] Astone P *et al* "Search for gravitational wave bursts by the network of resonant detectors" 2002 *Class. Quantum Grav.* **19**
[7] Astone P and Pizzella G 2002 *Astrop. Phys* **16** 441
[8] Papoulis A 1991 *Probability, random variables, and stochastic processes, 3rd ed.* (Singapore: McGraw-Hill International Eds.) p 247