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120 $\hbar$ SQUID amplifiers with a high-*Q* resonating input load

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Abstract

We report on the behaviour of a low-noise two-stage SQUID amplifier, which is strongly coupled to high-*Q* electrical resonators operating at audio frequencies. The behaviour of the system has been made stable, without adding noise, by means of a damping network. Thanks to the very high quality factor of the resonator, it is possible to evaluate the back action noise of the SQUID amplifier. The additive noise is evaluated as usual by measuring the open input noise. From measurements of back action and additive noises, performed in the temperature range 1.33 K–4.17 K and at 1.67 kHz and 11 kHz, the approximate noise temperature T_n' of the SQUID amplifier is evaluated and its minimum value corresponds to 120 quanta. Besides providing a full noise characterization of the SQUID amplifier, this system constitutes a simulator of a cryogenic resonant gravitational wave detector as regards operation frequency, quality factors and stability problem. On the basis of these results the operation of a resonant gravitational wave detector with an amplifier with energy sensitivity better than 100 $\hbar$ seems achievable.

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1. Introduction

Most of the operating cryogenic resonant gravitational wave detectors [1] employ as first amplifier a low-noise SQUID amplifier. Knowledge of the noise characteristics and the control of the dynamic input impedance of the SQUID amplifier are of fundamental importance for the sensitivity and stable operation of the detector for several reasons.

The first reason concerns the noise matching between the SQUID amplifier and the displacement transducer (the signal source) [2]. To optimize the impedance-matching network which is placed between the SQUID and the displacement transducer, it is necessary to know the noise characteristics of the SQUID amplifier and in particular the spectral densities S_i and S_v of the additive and back action noise sources I_n and V_n (see figure 1) [3, 4].

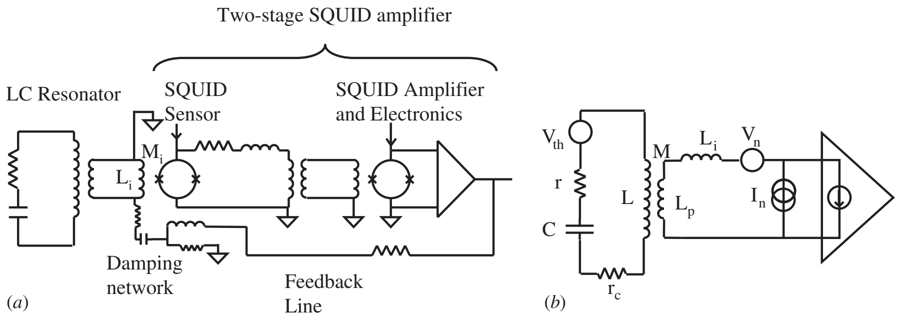

Figure 1. (*a*) Schematic circuit diagram of the two-stage SQUID coupled to the high-Q LC resonator. (*b*) The noise model of the resonator–SQUID system. The two-stage SQUID is modelled by an ideal current amplifier with the noise sources V_n and I_n.

Moreover, if the mechanical losses of the bar and transducer and the electrical losses of the matching network are low enough, then the detector can approach the energy sensitivity limit of the resonant detectors (Giffard's limit), which is given by the noise temperature of the amplifier times the Boltzmann constant [2, 5]. As the approximate noise temperature[1] $T'_n = (S_i S_v)^{1/2}/2k_B$ depends on both the noise sources S_i and S_v of the SQUID amplifier, only a detailed knowledge of the SQUID noise permits evaluation of the detector sensitivity limit in the case where the noise sources of the detector are only the thermal noise sources of the bar, transducer and matching network and the noise sources of the SQUID amplifier.

Another important problem related to the use of SQUID with high-Q resonating input load (as the three-mode system constituted by the bar, the transducer and a resonating matching network), is the stability of the entire system. As is known, the dynamic input impedance of the SQUID [6] can have a negative real part which can drive the system to instability.

For these reasons we have realized a low-noise two-stage SQUID amplifier and we have operated it strongly coupled to a high quality factor electrical resonator. This system simulates the detector as regards operation frequencies (audio frequency range) and quality factor (of the order of 10^6) of the SQUID input load. In this configuration the approximate SQUID noise temperature can be evaluated and the resonator–SQUID stability problem is studied.

2. Experimental apparatus and noise measurement

In figure 1(*a*) a simplified circuit diagram of the two-stage SQUID coupled to a high-Q electrical resonator is shown. In the two-stage configuration [7] the output voltage signal produced by the first SQUID is not sent to a room temperature electronics but amplified by a second SQUID which produces the feedback flux to make the response of the system linear. This configuration is more complicated than the single-stage one but offers some advantages. First of all, the noise contribution of the room temperature electronics can be made negligible. Secondly, the effect of the electromagnetic interference picked up by the long cable between the cryogenic components and the room temperature electronics is reduced. Thirdly, as the main part of the noise is produced by the first SQUID, the noise of the two-stage SQUID is thermal, that is it scales with the temperature.

Both the SQUIDs are commercial[2]. The first SQUID is not modulated and its output voltage is fed to the second SQUID amplifier through a matching network. The second

[1] The approximate noise temperature T'_n is obtained for uncorrelated noise sources and constitutes an upper limit for the complete noise temperature $T_n = (1/2k_B)(S_i S_v - \mathrm{Im}(S_{iv})^2)^{1/2}$.

[2] Quantum Design, 11578 Sorrento Valley Road, Suite 30, San Diego, CA 92121-1311, USA.

SQUID is operated open loop and read out by a standard manufacturer electronics with a 500 kHz modulation scheme. The system is operated in conventional flux-locked loop, with the output signal from the amplifier electronics sent through a one-pole integrator and fed back to the first SQUID through a feedback coil. Both SQUIDs are then locked through the same loop. The noise performance of the system is optimized by adjusting the bias current of the first SQUID and the flux of the second SQUID by means of its modulation coil (not shown in figure 1(*a*)).

Two high-Q LC resonators [4], operating at 1.67 kHz and 11 kHz, have been used. Both are based on low-loss low-stray-capacitance superconducting coils (0.55 H and 17.6 mH) and on Teflon capacitors (19.1 nF and 23.7 nF). The coil, capacitor and SQUID are housed in three separate superconducting shields.

The stability has been achieved thanks to a damping network, composed of a capacitor in series with a resistor, between the SQUID input coil and the feedback coil [8]. This network permits us to counterbalance the effect of the dynamic input impedance of the SQUID and to keep the apparent quality factor of the resonator at the constant value of a few thousands without adding noise, that is realizing a cold damping.

The noise model for the resonator–SQUID system [9] (see figure 1(*b*)) considers the thermal noise source V_{th} with spectral density $4k_{\mathrm{B}}Tr$ due to the intrinsic losses of the resonator ($Q_{\mathrm{i}} = \omega_0 L_{\mathrm{r}}/r$) and the noise sources V_{n} and I_{n} of the SQUID amplifier. A noise-free resistor r_{c} is included in the model to take into account the effect of the real part of the SQUID dynamic input impedance and is responsible for the apparent quality factor ($Q_{\mathrm{a}} = \omega_0 L_{\mathrm{r}}/(r + r_{\mathrm{c}})$. The noise spectrum at the SQUID output has a Lorentzian peak at the resonator resonance frequency. The expected variance of the noise peak is

$$\sigma_{\mathrm{out}}^2 = \frac{Q_{\mathrm{a}}}{Q_{\mathrm{i}}} A^2 \left\{ \frac{k_{\mathrm{B}}T}{L_{\mathrm{r}}} + Q_{\mathrm{i}} \frac{S_{\mathrm{v}}}{4\omega_0} \left(\frac{M}{L_{\mathrm{t}} L_{\mathrm{r}}} \right)^2 \right\},$$

where the first component is due to the resonator thermal noise and the second to the SQUID back action noise, $L_{\mathrm{t}} = L_{\mathrm{i}} + L_{\mathrm{p}}$, $L_{\mathrm{r}} = L - M^2/L_{\mathrm{t}}$ is the coil inductance reduced by the coupling to the SQUID and A is the gain between the resonator current and the SQUID output voltage. If the coupling M between the SQUID and the resonator is strong enough and the Q_{i} of the resonator is high enough, the back action noise can be emphasized over the thermal noise. Given the back action noise spectral density S_{v} and the additive noise spectral density S_{i} (a simple open input SQUID noise measurement), the calculation of the approximate noise temperature of the SQUID amplifier is straightforward.

3. Results and discussion

We have performed measurements of the back action noise of the two-stage SQUID amplifier at 1.67 kHz [10] and 11 kHz [11] and at temperatures between 1.33 K and 4.17 K. From these measurements and from additive noise measurements (as an example, see [7]) the approximate noise temperature data, shown in figure 2, have been obtained. The best approximate noise temperatures (63 μK at 11 kHz and 16 μK at 1.67 kHz) are, respectively, equivalent to an energy resolution $\varepsilon = k_{\mathrm{B}}T_{\mathrm{n}}'/\omega_0$ of 120 $\hbar$ and to 200 $\hbar$. The intercept of the data at 1.67 kHz is different from zero probably because of a non-thermal $1/f$ component which is also present, with the same ratio, in the additive noise. At 11 kHz the non-thermal component is completely negligible and the approximate noise temperature of the SQUID amplifier scales with the temperature. The slopes are in agreement with the theory within about a factor of 2. In fact, from the values of the SQUID loop inductance $L_{\mathrm{SQ}} \cong 80$ pH and shunt resistance $R_{\mathrm{S}} \cong 2\ \Omega$ of the first SQUID given by the manufacturer, the expected energy resolution

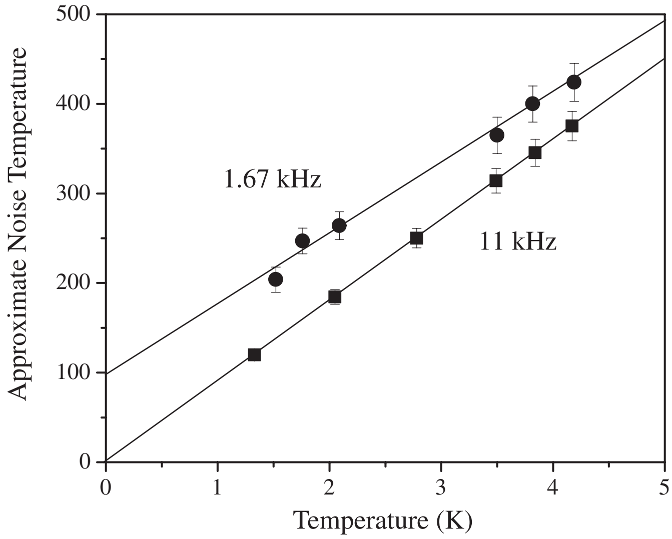

Figure 2. Approximate noise temperature of the two-stage SQUID amplifier expressed in equivalent number of quanta ($k_B T'_n/\omega_0\hbar$) as a function of the operation temperature at 1.67 kHz and at 11 kHz.

$\varepsilon = k_B T'_n/\omega_0 \cong 6.6(L_{SQ}k_B T/R_S) \cong 35\,\hbar T$. Probably this disagreement is not due to some inaccuracy in the values of the SQUID parameters used to calculate the expected approximate noise temperature but to a magnetic noise source near or in the SQUID chip. The fact that the disagreement of a factor of 2 is found in both the additive and back action noises supports this hypothesis [4]. The metal pads for the wiring connection of the SQUID chip or the shunt resistances themselves could represent possible sources of magnetic noise.

Work is in progress to measure the noise temperature of the two-stage SQUID down to 100 mK where the expected energy resolution is about $10\,\hbar$ at 11 kHz and $100\,\hbar$ at 1.67 kHz.

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