# Validation Of Data In Operating Resonant Detectors

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**Abstract.** Assessing the confidence of detection for candidate signals of gravitational waves is a particularly subtle matter. A fundamental step toward this achievement is the validation of the output data of the detectors involved. Here we present how this is accomplished in the operating resonant detector AURIGA by discriminating between satisfactory and unsatisfactory periods of operations on the basis of data self consistency. In particular, the statistics of the operating noise is checked against its simple model and the compliance to the expected shape of the candidates for burst gravitational wave events is assessed by means of a $\chi^2$ test. This approach helps in reducing the false alarm rate of each operating detector and moreover ensures the correctness of the estimated parameters of the candidate events.

## INTRODUCTION

The operating gravitational wave observatory made by the five resonant bar detectors ALLEGRO[1], AURIGA[2], EXPLORER[3], NAUTILUS[4] and NIOBE[5] is currently searching for burst signals by means of a time coincidence analysis between the candidate events provided by each detector under the International Gravitational Event Collaboration[6]. The confidence of detection relies mainly on the reduction of the false alarm rate to low values. This is helped by a careful determination of the time periods of satisfactory operation of each detector and of the compliance of the candidate signals to the expected shape. A future step to get the signature of single candidates of gravitational wave detection will require the measurement of peculiar properties of the gravitational wave, such as its propagation speed, the source location and the transversality and tracelessness of the Riemann tensor. New capabilities of the bar detectors –such as the submillisecond timing resolution[7] and the $\chi^2$ test on the shape of candidate events[8]– can play a crucial role in ensuring the confidence of detection.

In this paper we will present how the AURIGA detector is able to discriminate between satisfactory and unsatisfactory time periods of operation by testing the statistics of the measured noise and the self-consistency of the data analysis. We will briefly review the relevance of a statistical test of the compliance of each candidate signal to the expected shape from the viewpoint of validating the estimated parameters

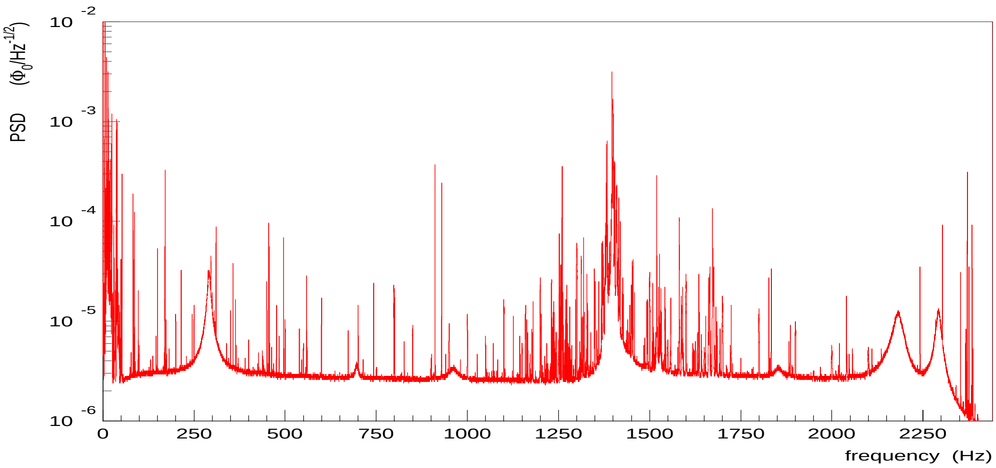


**FIGURE 1.** Noise power spectral density of AURIGA raw data averaged over 1 hour. The two peaks at 911 Hz and 930 Hz which dominates the spectrum around 1 kHz are the bar-transducer modes. The white noise level comes from the d.c. SQUID noise amplifier, corresponding to an energy resolution of about 4000 $\hbar$.

of the signal. Finally, we point out the near future opportunities given by the measurement of the arrival time of the g.w. bursts in the array of detectors and by testing the consistency of candidate g.w. coincidences with respect to the detected amplitude parameters at different detectors.

## DATA ANALYSIS AND NON STATIONARY BEHAVIOUR

The system of data acquisition and analysis of the AURIGA detector has been recently presented[9]. Let us recall here that the data acquisition is based on a signal sampling at 4.9 kHz and is synchronized to the Universal Time Coordinate within 1 μs by means of Global Positioning System clock. Fig.1 shows a sample of the raw data noise power spectrum for the year 1999; the bar-transducer mechanical resonances sensitive to the g.w. signals, show up as the highest peaks present in the 1 kHz region, respectively at 911 Hz and 930 Hz. The noise performance for impulsive signals recently achieved by the AURIGA detector is presented elsewhere in these proceedings[10] and corresponds to a minimum detectable Fourier transform of the g.w. amplitude of about $2\times10^{-22}$/Hz and a minimum detectable energy of 1 mK. The full raw data are archieved to allow for data reprocessing.

The AURIGA data analysis is fully numerical and is based on a Wiener-Kolmogoroff (WK) filter matched to δ-like signals. This filter assumes a very simple model for the noise of the detector in the bandwidth close to the bar modes, i.e. that the system is described by two coupled harmonic oscillators plus a white noise contribution from the redout SQUID amplifier. The noise spectral density can be written down as

$$S(\omega) = S_0 \prod_{k=1}^{2} \frac{(i\omega - q_k)(i\omega + q_k)(i\omega - q_k^*)(i\omega + q_k^*)}{(i\omega - p_k)(i\omega + p_k)(i\omega - p_k^*)(i\omega + p_k^*)}. \quad (1)$$

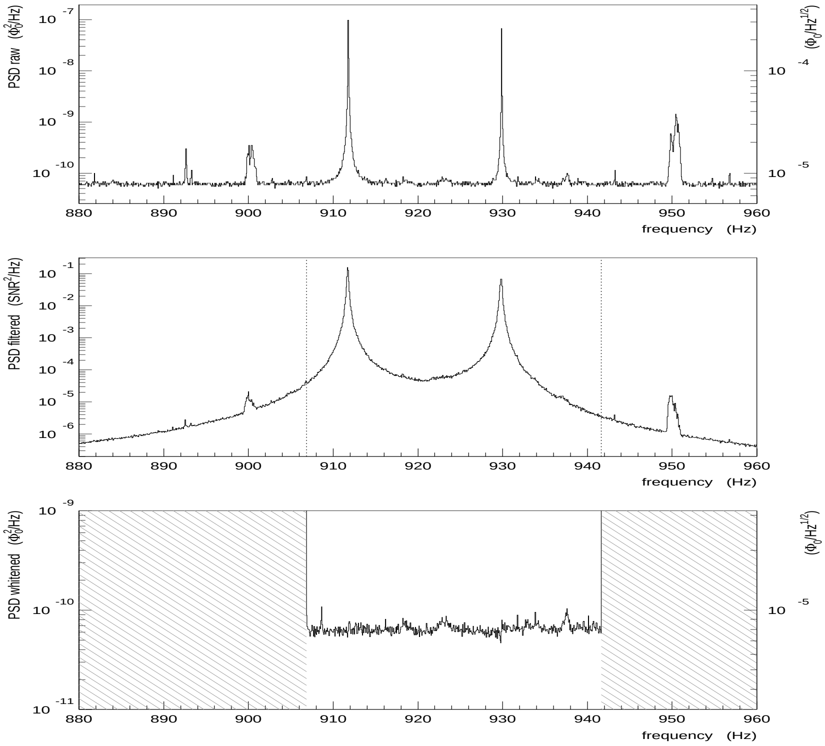


**FIGURE 2.** Noise power spectral density around the bar-transducer modes of AURIGA for: 1) raw data (upper); 2) W.K. filtered data (middle); 3) whitened decimated data (lower) for the same time period.

where the poles *p*'s and zeros *q*'s are 4 complex parameters, while $S_0$ is a (positive) real number. These parameters have to be estimated to build the WK filter, but due to the unavoidable non-stationarieties of the system some of their values changes significantly in time. If the non-stationarity is slow, i.e. occurs on a time scale longer than the relaxation time of the modes, the data analysis has to track them by a slowly adaptive algorithm.

We use different techniques to estimate each parameter:

■ $\omega_k \equiv \mathrm{Im}(p_k)$ the mode's frequencies are monitored by two fully digital lock-ins.

■ $\Delta\omega_k \equiv \mathrm{Re}(p_k)$ the pre-detection bandwidth is known not to be a critical parameter, and is left to the value measured at the beginning of a data taking period

■ $\omega_k^{opt} \equiv \mathrm{Im}(q_k)$ the post-filtering frequency is practically identical to the pre-filtering one

■ $\Delta\omega_k^{opt} \equiv \mathrm{Re}(q_k)$ the post-filtering bandwidth is adapted so to keep flat the whitened data spectrum (see fig. 2*d* in ref. 11, these proceedings)

■ $S_0$ the level of the amplifier's white noise is monitored by a lock-in displaced from the detector modes.

In practice, this model works well within a reduced bandwidth of about 35 Hz around the bar-transducer modes. The frequency domain expression for the whitening filter and for the W.K. filter matched to δ-like signals are ratios of polinomials as well, as shown elsewhere in these proceedings[11]. The actual implementation of the data filtering in AURIGA is however made in the time domain[9] by means of an Auto-

Regressive Moving Average. The whitening filter is applied only after the bandwidth of the data stream is narrowed around the bar-transducer modes where the noise model proves to work and, in the presence of noise alone, the whitened data in the reduced bandwidth do show a white noise power spectrum (see Fig. 2).

To track the slow non-stationarieties we use moving averages to smooth the parameters estimates over time scales of the order of the system's proper relaxation time $(\Delta\omega_k)^{-1}$~1000s, much longer than the Wiener filter time $(\Delta\omega_k^{opt})^{-1}$~1s. A non stationary behaviour faster than $(\Delta\omega_k)^{-1}$ does not allow to estimate correctly the noise parameters, and therefore the analysis should point out this difficulty to the experimentalists, as discussed in the next section. A possible signal would instead show up as a very fast variation of the detector output noise, limited in time and not too long with respect to the Wiener time.

When a signal affects the noise the whitened data show a residual color even if the parameters estimates are correct, so the estimates of the noise parameters cannot be trusted, as is shown elsewhere in these proceedings[11].

## SATISFACTORY AND UNSATISFACTORY DETECTOR OPERATION

A first level of vetoes on time periods of detector operation is set by the experimentalists, who judge what operations are known to affect the detector output, namely some of the manteinance operations, calibration procedures, failures and so on. These vetoed periods can never be considered as useful observation time of the detector. The AURIGA data analysis provides also for automatic vetoing of time periods in which the statistics of the observed noise is not as expected. In particular, the analysis tests if the filtered and whitened data follow a gaussian statistic and checks if the whitened data are uncorrelated. When the noise is not found to be as expected, then the analysis marks that time period as one affected by “excess noise” or overimposed “signals”. In case this condition occurs too frequently, we conclude that the noise model to which the filter is matched is not the right one and consequently that the filtered data of the detector are unsatisfactory. These time periods are vetoed from the observation time of the detector unless some new filtering attempt is succesful in better matching the measured noise.

In more detail, the analysis groups the data streams of both filtered and whitened data in 2 minutes long buffers. The analysis tests if the statistics of each buffer is gaussian buffer by buffer, both by looking at higher moments of the distribution, namely the Kurtosis, and by performing a Chauvenet selection. The latter procedure consists in eliminating those data samples which fall outside 3 times the calculated root mean square value of the buffer, then iterating the same procedure until no other data sample is eliminated. A data buffer is considered to be gaussian if its Kurtosis is compatible with that obtained in Monte Carlo simulatons within 99.7% Confidence Level and if the Chauvenet algorithm converges within a few steps eliminating at most a few percent of the data samples. Moreover, the whitened data buffers whose correlation exceeds a 99.7% CL threshold with respect to Monte Carlo simulations are not used for the estimation of the noise parameters.

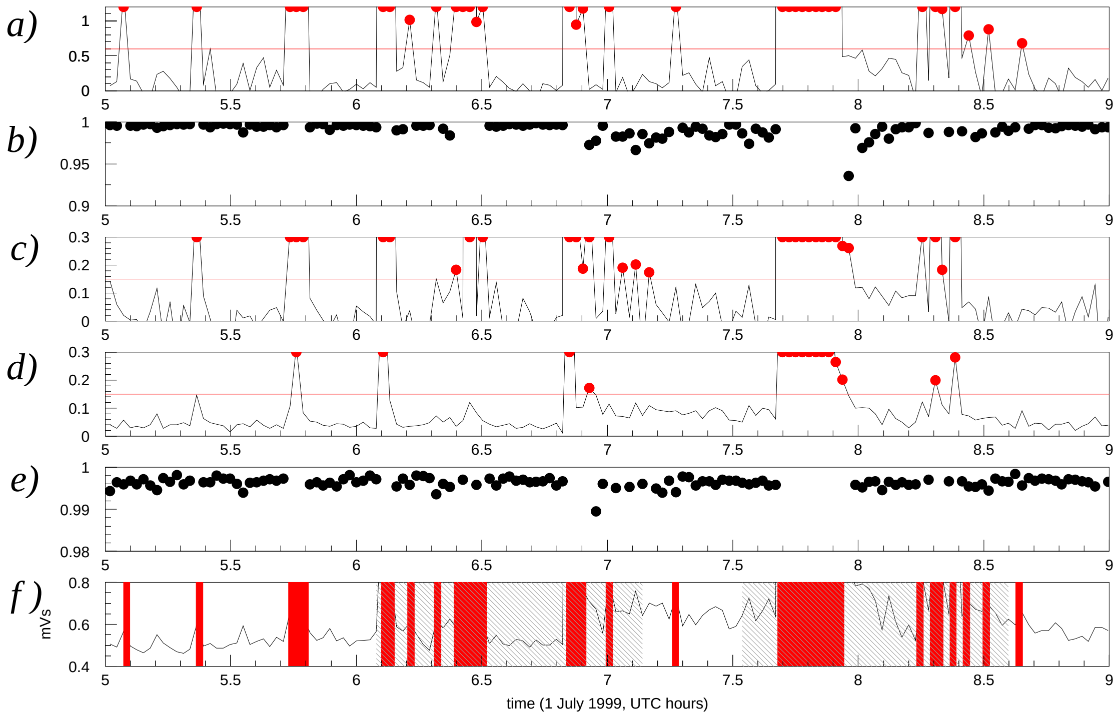


**FIGURE 3.** The statistical tests performed on filtered and whitened data in four hours of AURIGA operation. For each buffer are shown the Kurtosis and the fraction of data kept after the Chauvenet selection. Filtered data: a) Kurtosis, b) Chauvenet selection. Whitened data: c) Kurtosis, d) correlation, e) Chauvenet selection. Thin horizontal lines in a), c) and d) stand for the 3 sigma threshold on estimates as calculated from the modeled statistical fluctuations. The bottom graph f) shows: 1) the standard deviation of the filtered data buffers (thin line); 2) the time periods corresponding to data buffers with bad statistical properties (gray shadows); 3) the vetoed periods of operation due to too frequent bad data buffers (dashed areas).

The discrimination between good and bad data buffers by requiring strict compliance of the noise with a parametric model is a cornerstone feature of the AURIGA data analysis. Sample pictures of good and bad buffers are reported elsewhere in these proceedings[11]. Fig.3 shows the result of this procedure for four hours of AURIGA operation. When either a filtered data or a whitened data buffer is bad, the corresponding time period is not used to estimate the noise parameters of the detector and the W.K. filter is "freezed" to the previous condition. The bottom graph of Fig. 3 shows bad buffers times as gray areas.

Two main situations may then arise: either there is a dominant contribution of the modeled quasi-stationary noise with short time periods showing unmodeled excess noise and/or signals, or the data are dominated by unmodeled excess noise. In the first case, the bad buffers are rare enough so that the analysis is able to reliably estimate the noise parameters and to adapt the W.K. filter to any slow non-stationary noise behaviour by using good buffers only. The estimate of the noise parameters is therefore performed in a reliable way by using only the periods when the modeled noise is dominating and disregarding the bad buffers, which instead contain some "signals".

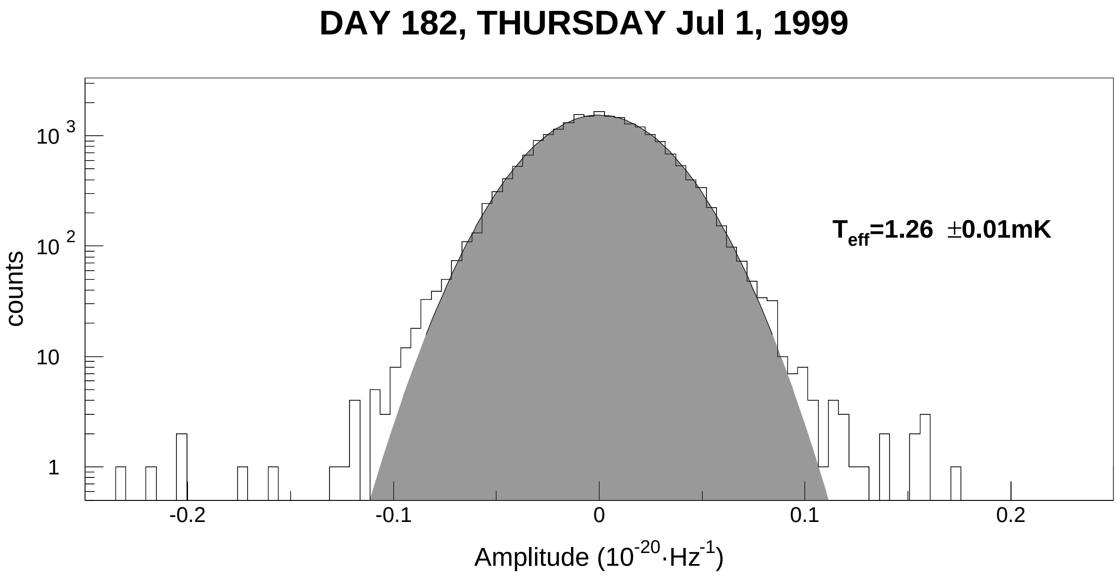


**FIGURE 4.** Amplitude histogram of filtered data during one day of satisfactory operation of the AURIGA detector. The data has been subsampled one per second to get almost independent samples. The small deviations from gaussian statistics at high amplitudes are due to the presence of signals overimposed on the noise.

This is therefore a satisfactory condition for detector operation, corresponding to the time periods in Fig. 3 where the bottom graph is not dashed. Fig.4 shows a sample amplitude histogram of the filtered data during the satisfactory operation of the detector during the same day of Fig. 3; the statistics is gaussian with small excess counts in the tails due to the “signals” present in the rare bad buffers.

The other main operating condition, i.e. that the data are dominated by unmodeled excess noise, occurs when at least half of the buffers are found to be bad within a fixed time window, ten buffers long in our case. In this condition, the W.K. filter is badly matched to the noise –in fact it could be that WK linear filter theory is not applicable. The analyzed data therefore lack of self-consistency and the output data is vetoed, as shown in the bottom graph of Fig.3 with dashed areas. Under this condition the detector is not necessarily blind and with different noise models and/or parameters one could recover some of the vetoed observating time.

## GOODNESS OF THE FIT TESTS AND COMPLIANCE OF CANDIDATE SIGNALS WITH THE EXPECTED SHAPE

Once that the W.K. filtered output is validated as described above, the compliance of the shape of each candidate signal to the one that the W.K. filter was looking for is to be discussed. By definition, during satisfactory AURIGA detector operation the noise statistic is purely gaussian, so the W.K. filter is a maximum likelihood fit[13] and the goodness of the fit must be checked by means of a $\chi^2$ test. In all the operating resonant bar detectors, the filter is matched to $\delta$-like g.w. and the fitting parameters are the wave amplitude and arrival time. In particular, the best estimate of event amplitude is given by the local maximum value of the interpolated W.K. filter output and its corresponding time coordinate is the best estimate of the arrival time.

In the AURIGA filtered data, a δ-like g.w. would show up as a beating signal sampled at 4.9 kHz, with a carrier frequency centered between the bar-transducer modes and with a rise and fall time given by the Wiener time, as is shown elsewhere in these proceedings[11]. The sampled signal is reconstructed in the continuum to search for the local maximum within 3 Wiener time by a max-hold algorithm. Then the mean square differencies are computed between the sampled data and the template function evaluated at each sample, and this number is used as a test statistic. In fact, it is just the standard $\chi^2$ test. The details of its implementation in the AURIGA data analysis is described in ref. 8. Such a test provides a mean to discriminate between candidate g.w. bursts exciting the bar and other kinds of spurious excitations entering the detector at different ports and/or not δ-like shaped. Here let us state its relevance from the data validation point of view.

As the WK filter theory depends on the compliance of both the noise and the signal with the models, it is clear why it is so important to have checked the first to make accurate statements on the second. For a candidate event in a satisfactory AURIGA operation, passing the $\chi^2$ test is a necessary and sufficient condition for assessing the reliability of the estimates of the event parameters. If the test fails we reject these estimates as biased and state there was no event with the expected shape. Of course, a signal whatever was there indeed, and we can infer from it a template to build a new filter, in order to find out if similar events has been or will be present in the detector output. An efficient way to perform this task is to project the signal on a set of noise autocorrelation matrix eigenfunctions and store just the coefficients of the chief terms of the expansion[12].

An overall consistency check of the detector operation can be realized by applying the $\chi^2$ test to mechanical calibration signals applied to the bar, which are anyway a natural step to complete the calibration procedure. As for the AURIGA detector the work is in progress and up to now we tested the system in this definitive way only from the acquired data on, that is by providing software calibration signals overimposed to the real raw data stream after the ADC.

Fig. 5 shows the histogram of the calculated $\chi_e^2$ for the events exchanged by AURIGA under the IGEC Collaboration from Sept. 1997 to March 1998. AURIGA exchanged all found events with Signal to Noise Ratio (SNR) in amplitude larger than 5 and with a 141 degrees of freedom calculated $\chi_e^2$ smaller than 1.5, corresponding to a confidence level of about $1 - 1\times10^{-4}$. The effective distribution of the calculated $\chi_e^2$ of the events was found to be in agreement with the $\chi_r^2$ distribution at least for the low SNR events which are the great majority of the exchanged ones. Most of the events in Fig.5 fall inside the expected $\chi_r^2$ distribution as well.

We can point out another useful application of goodness-of-the-fit tests[8] for the currently operating array of detectors. Since all the detectors are almost parallel and, apart from NIOBE, their operating frequencies fall within a 40 Hz bandwidth around 900 Hz, the amplitudes of g.w. signals detected in a N-fold coincidence must be consistent. If each detector implements an analysis capable of discriminating the

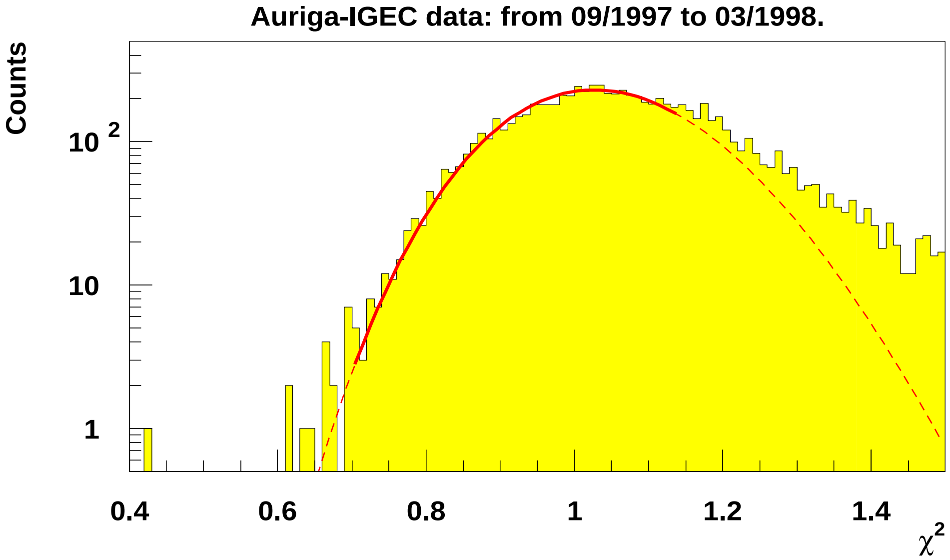


**FIGURE 5.** Histogram of the calculated $\chi_e^2$ of the candidate events of the AURIGA detector under the IGEC Collaboration from Sept. 1997 to March 1998. These 15854 events have amplitude SNR>5 and are below a $\chi_e^2$ threshold which corresponds to a false dismissal of about $1\times10^{-4}$. The $\chi_r^2$ distribution is also shown as a continuous line.

satisfactory periods of operation as AURIGA does, then the amplitude estimate $A_i$ of the i-th detector follows a gaussian distribution of variance $\sigma_i^2$ and

$$A = \frac{\sum_{i=1}^{N} \frac{A_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \quad , \quad \chi_A^2 = \sum_{i=1}^{N} \frac{(A_i - A)^2}{\sigma_i^2} \qquad (2)$$

are respectively the optimal amplitude estimate for the g.w. and the corresponding value of the experimental $\chi^2$ with N-1 degrees of freedom.

As an exercise to probe the efficiency of this $\chi_A^2$ test we randomly grouped the AURIGA events of Fig.5 in random triplets and quintuplets. The result is shown in Tab. 1. The relatively low fractions of rejected random coincidences are likely a lower limit of the method, because AURIGA showed a stable noise performance in that time period and therefore most of its exchanged events are close to SNR=5 and do have the same amplitude within the proper $\sigma$. The efficiency of rejection should increase significantly for higher SNR events and/or in the case that the detectors are not setting the same amplitude thresholds. Especially in dealing with high SNR events, however, care must be taken to account for inaccuracies of the detector calibrations.

**TABLE 1. Implementation of $\chi_A^2$ on randomly chosen triplets and quintuplets from AURIGA candidates events.**

| Confidence Level | Random triplets Fraction rejected | Random quintuplets Fraction rejected |
|---|---|---|
| 0.9 | 0.23 | 0.30 |
| 0.99 | 0.11 | 0.17 |

This goodness-of-the-fit method can be generalized to the case of detectors with different antenna patterns at the cost of solving also for the two parameters describing the source location, unless they are known by other means. It has recently been proposed also a different approach aimed at testing amplitude consistency in 2-fold coincidences between parallel detectors[14]. The approach consists in checking how the value of the ratio of the amplitudes of the events in coincidence compares with the distribution of the values calculated for spurious coincidences, as those generated by time shifting the response of one detector with respect to the other.

## FINAL REMARKS

We have shown how a satisfactory level of data validation can be accomplished for an optimal WK filter operating on a real detector. The key point is the ability to check the compliance of the filter with the noise of the detector and use this test to discriminate between satisfactory and unsatisfactory time periods of detector operation. In the satisfactory periods the filter performance is validated and goodness-of-the-fit tests can be performed to check also the compliance of the observed signals with the signal shape to which the filter is matched, therefore assessing the reliability of the estimates of the signal paramenters.

The confidence of signal detection would be further improved if some peculiar properties of the incoming gravitational waves can also be detected. The capability of measuring arrival times with submillisecond resolution has been demonstrated on a resonant bar prototype[7] and would provide, once implemented on improved versions of the operating detectors with a larger post-filtering bandwidth, for the measurement of the wave propagation speed and of the source location[15].

## ACKNOWLEDGMENTS

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