Dc superconducting quantum interference device amplifier for gravitational wave detectors with a true noise temperature of 16 μK

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We report on the noise characterization of a two-stage dc superconducting quantum interference device (SQUID) amplifier developed for resonant gravitational wave detectors. The back action noise is estimated by coupling the SQUID to an electrical resonator at 1.6 kHz with $Q=1.1\times10^6$. From measurements of back action and additive SQUID noise, performed in the temperature range 1.5–4.2 K, an upper limit is set on the noise temperature T_n of the device at the resonator frequency. The best value obtained at 1.5 K is $T_n \leqslant 16$ μK and corresponds to 200 resonator quanta. The thermal component of the noise temperature is found in reasonable agreement with the predicted value. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1408276]

A coupled dc superconducting quantum interference device (SQUID) is often modeled [Fig. 1(b)] as an ideal current amplifier with two noise sources: a current noise source I_n in parallel with the input port that provides for the additive noise, and a voltage noise source V_n in series with the input port that acts as a back action generator on the input circuit. In general, the most appropriate figure of merit of the device is the noise temperature T_n, defined by

$$2k_bT_n=[S_{vv}S_{ii}-\mathrm{Im}(S_{iv})^2]^{1/2}, \tag{1}$$

where k_b is Boltzmann's constant, S_{vv}, S_{ii} are the monolateral power spectral densities of the two noise generators, and S_{iv} is the cross-correlation spectral density, which, according to Clarke–Tesche–Giffard (CTG) theory,[1,2] is purely imaginary. The same theory predicts that the noise temperature of a dc SQUID, at a frequency ω in the white-noise region and at temperature T, is given by

$$T_n=\gamma T\frac{\omega L_S}{R_S}, \tag{2}$$

where R_S and L_S are, respectively, the shunt resistance and self-inductance of the SQUID and γ is a numerical factor: γ=2.8 by applying definition (1) and γ=6.6 if the correlation term S_{iv} is neglected. T_n is the temperature at which the thermal noise power of the input source equals that of the amplifier, provided that the source impedance is optimized. k_bT_n can also be interpreted as the minimum energy change of a harmonic oscillator that the amplifier can detect under optimum matching conditions, as in the case of a resonant gravitational wave (GW) detector.[3]

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The energy resolution, defined by $\epsilon=L_iS_{ii}/2$ and thus related only to the additive component of the SQUID noise, is usually reported as the figure of merit in most papers about dc SQUIDs. However a measurement of ϵ cannot be considered a complete noise characterization, because it does not take into account the back action noise, which is not a negligible effect in at least some high-sensitivity applications, such as radio-frequency amplifiers[4] or GW detectors.[5]

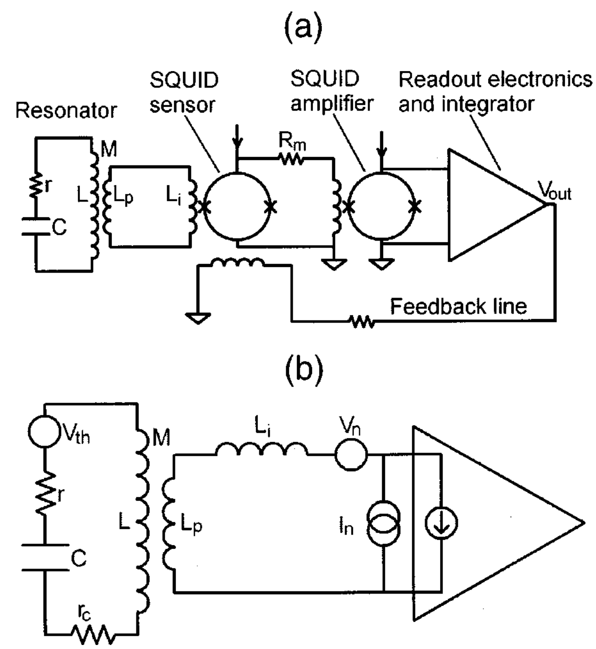

FIG. 1. (a) Schematic circuit diagram of the two-stage dc SQUID coupled to the high-Q resonator. (b) The dc SQUID is modeled by an ideal current amplifier with the noise sources V_n and I_n. The real part of the dynamic input impedance is represented for convenience by the cold resistor r_c in series to the resonator.

Measurements of the noise temperature have been performed on several SQUID systems, very recently even on a near-quantum-limited device at 0.5 GHz.[6] However, in the frequency range, around 1 kHz, which is of interest for resonant GW detectors, the predicted back action of the dc SQUID is very small. Indeed, it has been measured only on SQUIDs showing a back action artificially enhanced[7] or much bigger than expected[8] and on a commercial system with $\epsilon \approx 3000\ \hbar$.[2,9]

In this letter, we report the noise measurement on a two-stage SQUID system developed for the GW detector AURIGA,[5] which achieves the best measured noise temperature of an amplifier operating in the audio-frequency range, more than one order of magnitude lower than previous results.[8,9]

The measurement method is very similar to that described in Refs. 2 and 9. The SQUID is coupled to a high-quality factor electrical rLC resonator through a superconducting matching transformer (Fig. 1). The resonator is vibrationally and magnetically shielded enough to make excess noise negligible, so that only two modeled sources of voltage noise act on the resonator: the thermal source $V_{\rm th}$ associated with resonator intrinsic losses, with spectral density $4k_b$Tr, and the SQUID back action generator V_n. The input impedance of the SQUID is represented by a pure inductance L_i. A noise-free resistor r_c is added to the model to take into account the effect of the real part of the SQUID dynamic input impedance[10] or the effect of additional feedback loops that can be employed to realize a cold damping of the resonator.

Following the model we find that around the resonance frequency ω_0, provided that the quality factor $Q \gg 1$, the contribution of the additive SQUID noise is negligible, and the total current noise power spectral density flowing in the input coil is a classical Lorentzian curve:

$$(S_{ii})_{\rm tot}(\omega) = \frac{a(\omega_0)}{\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2+\left(\frac{\omega_0}{\omega Q}\right)^2}, \tag{3}$$

where

$$a(\omega_0) = \left(\frac{M}{L_t}\right)^2 \frac{4k_BT}{\omega_0 L_r Q_i} + \left(\frac{M}{L_t}\right)^4 \frac{S_{vv}(\omega_0)}{(\omega_0 L_r)^2}, \tag{4}$$

and [see Fig. 1(b)] $L_t = L_i + L_p$, $L_r = L - M^2/L_t$ is the coil inductance reduced by the coupling to the SQUID, $\omega_0 = (L_rC)^{-1/2}$ is the resonant angular frequency, $Q = \omega_0 L_r/(r+r_c)$ is the overall quality factor, and $Q_i = \omega_0 L_r/r$ is the intrinsic quality factor. From a measurement of $a(\omega_0)$, it is possible to estimate the back action noise spectral density $S_{vv}(\omega_0)$, provided that Q_i and M/L_t are high enough to make the back action term in Eq. (4) not negligible with respect to the thermal term.

Experimental details on the dc SQUID are reported in a preliminary work.[11] It is a two-stage system [Fig. 1(a)] where a second SQUID (the amplifier) is used as a low-noise preamplifier of the first (the sensor). Both sensor and amplifier chips were manufactured by Quantum Design.[12] The sensor parameters given by the manufacturer are $R_S = 2\ \Omega$ and $L_S = 80$ pH. The SQUIDs are placed in different shields and are connected to the room-temperature electronics through different cables in order to avoid any stray cross talk between the wires. The SQUID sensor, biased through a battery-powered current box, is not modulated and its output voltage is fed through a matching resistor $R_m = 2.2\ \Omega$ to the SQUID amplifier, which is finally read out by standard manufacturer electronics with a 500 kHz modulation scheme. The system is operated in a conventional flux locked loop, with the output signal from the amplifier electronics sent to a one-pole integrator and fed back to the SQUID sensor. The maximum bandwidth of the system in closed-loop mode is limited by various filtering stages to about 50 kHz. The slew rate is about $2\times10^4\ \ \phi_0$/s.

The high-Q resonator[2,9,13] is based on a low-loss, low-stray-capacitance superconducting coil with a measured inductance (in its housing) $L = (0.554\pm0.005)$ H, and a Teflon capacitor $C = (19.1\pm0.1)$ nF. The coil, the capacitor, and the SQUID are housed in three separate superconducting shields. The SQUID is coupled to the resonator by means of a flux transformer whose elements are the SQUID input coil $L_i = 1.64\ \mu$H and a pickup coil $L_p = 2.8\ \mu$H tightly coupled to the main coil L with a geometrical coupling factor $k = M/(LL_p)^{1/2} \approx 0.5$. A weak capacitive coupling between feedback line and input coil, not shown in Fig. 1, is employed to increase r_c and damp the resonator; the quality factor is decreased from its intrinsic value $Q_i \approx 10^6$ to $Q \approx 10^3$. This trick helps to avoid the well-known instabilities[10] of the SQUID–high-Q resonator systems.

For the noise measurement, the flux and bias current working point of the SQUID sensor is first optimized by minimizing the out-of-resonance broadband noise. Then, the output voltage noise is sampled and fast Fourier transformed (FFT) through a computer-based spectrum analyzer. The frequency resolution is chosen to be at least ten times smaller than the bandwidth of the resonator, and up to 200 periodograms are averaged. A weighted nonlinear fitting procedure is then applied to the resulting averaged spectrum by using Eq. (3) as the fitting function in a 20 Hz window around ω_0 where no spurious peaks are present. The relative uncertainty on the data is taken as $1/N^{1/2}$, where N is the number of FFT that are averaged. The values of $a(\omega_0)$, ω_0, and Q are extracted as fitting parameters, and a χ^2 test with a 95% confidence level is performed to check the goodness of fit.

$M/L_t = 133.5\pm0.5$ and $L_r = (0.475\pm0.002)$ H were measured during a separate calibration run where the capacitor was replaced by a known resistor, whose Nyquist noise was used as the calibration signal. The intrinsic quality factor at $\omega = \omega_0$ was found to be $Q_i = (1.08\pm0.01)\times10^6$ by measuring the time decay constant of the resonator in another separate run when the SQUID was weakly coupled to the resonator in order not to affect the quality factor. The resonance frequency during all measurements was $\nu_0 = \omega_0/2\pi = 1671.45$ Hz. Knowing all these parameters, the intrinsic thermal noise of the resonator at a given temperature, given by the first right term in Eq. (4), can be evaluated.

In Fig. 2, the measured value of $a(\omega_0)$ is shown as function of temperature in the range 1.5–4.2 K. The calculated intrinsic thermal noise of the resonator is also shown; a previous series of tests,[2,13] realized on the same experimental apparatus by varying ω_0 and Q_i, showed that the thermal

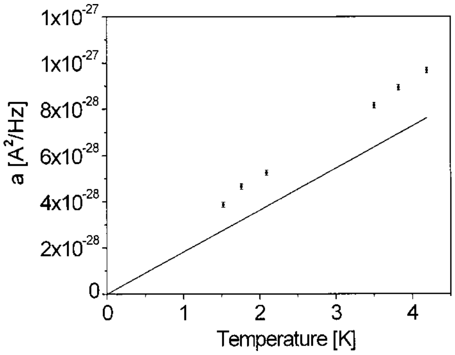

FIG. 2. Measured resonator noise expressed by the quantity $a(\omega_0)$ as function of temperature at $\nu_0=1671$ Hz. The continuous line represents the calculated resonator thermal noise.

noise is in good agreement with the prediction of the fluctuation–dissipation theorem in the range 1.5–4.2 K. After subtracting the thermal noise from the measured data in Fig. 2, the SQUID back action contribution, and hence $S_{vv}(\omega_0)$, are computed through Eq. (3). In principle, the power spectral densities S_{ii} and S_{iv} could be measured simultaneously to S_{vv} by means of a more complex fitting procedure over the whole bandwidth, about 200 Hz around the resonance for the present experimental setup, where the resonator noise is dominant over the SQUID additive noise. Unfortunately, too many vibrational peaks are, for now, present in that region, making the fitting procedure rather difficult. For this reason the additive current noise $S_{ii}(\omega)$ is measured as a function of T in a separate run when the input coil is left open, so that the back action voltage noise gives no contribution.[11] The approximate noise temperature at $\omega=\omega_0$, $T_n'=[S_{vv}(\omega_0)S_{ii}(\omega_0)]^{1/2}/2k_b$ is then computed and plotted in Fig. 3 in units of temperature and number of quanta $k_bT_n'/\hbar\omega_0$. T_n' differs from T_n in the fact that it does not consider the cross-correlation spectral density S_{iv}, which

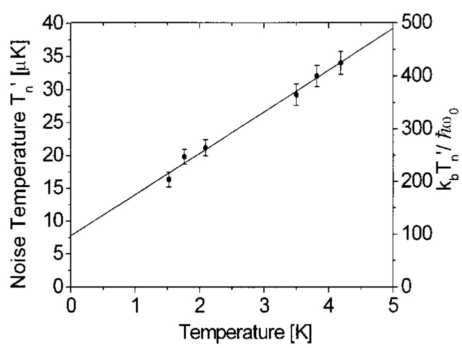

FIG. 3. Approximate noise temperature $T_n'=[S_{vv}(\omega_0)S_{ii}(\omega_0)]^{1/2}/2k_b$ at $\nu_0=1671$ Hz. The continuous line is the best linear fit to the data.

is not estimated in our experiment. However, the relation $T_n'\geqslant T_n$ follows immediately from Eq. (1), so that T_n' gives, in any case, an upper limit on the true noise temperature T_n.

The best linear fit on experimental data in Fig. 3 gives $T_n'=(7.7\pm1.5)\times10^{-6}$ K$+(6.3\pm1.0)\times10^{-6}$ T. The intercept corresponds to (95 ± 20) quanta and is probably related to a $1/f$ contribution. This hypothesis is difficult to check, because S_{vv} is measured only at the fixed frequency ω_0, but can be supported by the estimation of the low-frequency component of S_{ii}, since the approximate relation $\epsilon\approx k_bT_n'/\omega_0$ is expected from CTG theory. We find indeed that $1/f$ noise does not depend substantially on temperature and that its contribution to ϵ at $\omega=\omega_0$ is $(40\pm8)\ \hbar$, thus of the same order of magnitude of the measured intercept in Fig. 3. The contribution to T_n' of the room-temperature electronics, which is also temperature independent, is estimated to be smaller by more than one order of magnitude. Regarding the slope of the linear fit in Fig. 3, it is higher than the value predicted by Eq. (2) by a factor 2. This level of agreement between CTG theory and experiment is typical,[7] so we believe the device to be rather close to its intrinsic noise.

Finally, we remark that the minimum measured upper limit on the noise temperature, obtained at 1.5 K, is $T_n'=16$ μK. It corresponds to a minimum detectable energy of, at most, 200 quanta in a resonant GW detector operating in the range of frequency around 1 kHz.

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