Wideband dual sphere detector of gravitational waves

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We present the concept of a sensitive *and* broadband resonant mass gravitational wave detector. A massive sphere is suspended inside a second hollow one. Short, high-finesse Fabry-Perot optical cavities read out the differential displacements of the two spheres as their quadrupole modes are excited. At cryogenic temperatures one approaches the Standard Quantum Limit for broadband operation with reasonable choices for the cavity finesses and the intracavity light power. A sapphire detector of overall size of 2.6 m, would reach spectral strain sensitivities of 10^{-23} Hz$^{-1/2}$ between 1000 Hz and 4000 Hz.

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Resonant mass detectors of gravitational waves (GW) are commonly indicated as narrowband devices. In currently operating cylindrical bar detectors [1], all equipped with resonant transducers, the bandwidth, even in the limit of quantum limited performance of the final displacement readout, would not open up for more than about the frequency interval between the resulting two mechanical modes of resonance, currently about 30 Hz.

To look for the widest possible bandwidth is roughly equivalent to maximize the signal to noise ratio (SNR) for impulsive signals. A general analysis of the problem, valid for any *linear* detector, has been given by Price [2]. Secondary *resonant* masses are linked to the main resonant mass to efficiently couple the signal energy to the final readout, but the bandwidth is limited to a fraction of the main resonator frequency. Such a coupling is poorer the smaller the number n of secondary resonators, and correspondingly the bandwidth increases with (the square root of) n. The inevitable conclusion is that cryogenic resonant mass GW detectors have narrow band, if one does not match the final readout with a large number of secondary resonators [2].

This stems however from the noise performance of the readout systems considered at the time. In fact, if a *single* mechanical resonator were driven only by its thermal noise and by a signal force, with negligible readout noise, the SNR would be *independent* of frequency, and thus the band would open up. Of course this cannot be the case, even in principle, because the final amplifier noise cannot be less than its Standard Quantum Limit (SQL) [3,4]. As Scan be seen in reference [2], the problem of investigating the detector configuration giving maximal strain spectral sensitivity *and* bandwidth, for a single mechanical mode detector, comes to the practical problem of having the equivalent dimensionless amplifier resistance R_{eq}, in which the signal energy is ultimately dissipated, as large as possible. The readout systems considered in [2] would give $R_{eq}=10^{-5} \div 10^{-7}$, and the fractional bandwidth would be correspondingly small as $\Delta f/f = R_{eq}$. The alternatives considered at the time were the multimode systems, with $n = 2$ [5], and with $n > 2$ [6]; proposals involving *non*-resonant mechanical impedance match also appeared in the literature [2,5]. To date, only two-(mechanical) mode systems have worked their way into operating detectors.

We have been attracted by the possibilities offered by optical readout systems, as vigorously developed for interferometric GW detectors, and more recently applied in connection with cryogenic "bar" GW antennas [7,8]. We take a Fabry-Perot optical cavity as the motion sensor. In a system under development [8] the length of the sensor cavity is compared to that of a second cavity, separately kept, which acts as reference. We do not take into account here the noise introduced by the reference cavity, assuming for simplicity that it is negligible. With a sensor cavity length of the order of centimeters there is no loss of signal strength for finesses F as high as the highest attainable with current technology, $F = 10^6$, for GW in the kHz range. So we have considerable freedom to vary the finesse and the light power P incident on the cavity, in search for optimal conditions, which do not demand unreasonable values for these parameters.

That this is the case can be seen using the considerations of reference [2] and writing R_{eq} for an optomechanical SQL device, based on high finesse Fabry-Perot cavities:

$$R_{eq}(\omega) = \frac{1}{m\omega_1} B \frac{F^2 P}{\omega} \left[1 + \left(\frac{2FL_c\omega}{\pi c} \right)^2 \right]^{-1} , \qquad (1)$$

where m and ω_1 are respectively the resonator mass and angular frequency of resonance, L_c is the cavity length, and c the speed of light. B is a dimensional parameter, proportional to the square root of the photodiode quantum efficiency and to the light frequency, and depending on cavity mirrors' transmission coefficient and on light phase modulation depth; typically $B \simeq 4\cdot10^{-3}$ s/m^2 for a wavelength $\lambda = 1.064$ μm [9]. For resonator masses m in the range of a few tons, at frequencies $\omega \sim \omega_1 \sim 2\pi\,2$ kHz, it is possible to have R_{eq} as large as 10^{-1}, with $F \sim 10^6$ and $P \sim 10\ W$. As a consequence, it makes sense to consider a *non*-resonant optical readout for a resonant mass GW detector.

Let us then turn to the primary mechanical resonator, whose motion is directly related to the incoming GW. We take into consideration both solid and hollow spheres as resonant systems of interest. They are very attractive for a number of reasons, and in fact they have received significant attention in the literature of the last few years [10–13]. Spherical detectors are omnidirectional, have a more efficient coupling to the GW field relative to cylindrical bars, both in the first and in the second quadrupole mode, and enable a deconvolution of the GW signal if they are equipped with five (or more) suitable motion sensors [10,11,14]. For instance, a chirp signal from a merging compact binary can be fully deconvolved with a spherical detector [15]. Two spheres would make up for a complete observatory, in which not only direction and polarization, but also versus of propagation of the incoming wave can be resolved [16,17] —see also [18]. Located close to an interferometric detector, a spherical detector could be used for searches of stochastic background [19]. Such capabilities make of the spherical detector a conceptually unique device.

However, the sensitive spherical detectors proposed in the past suffer from the above discussed bandwidth limitation. As an example, a hollow sphere of CuAl$^{10\%}$, 4 meters in diameter and 30 centimeters thick, cooled at sub-Kelvin temperatures and equipped with *resonant* transducers and a quantum limited readout, gives a spectral strain noise as low as $6\cdot10^{-24}$ Hz$^{-1/2}$ [13], but only in two bands of 35 Hz and 135 Hz, respectively around the first and second quadrupole resonances at 350 Hz and at 1350 Hz, Fig. 3.

Let us then consider a spherical detector with *non* resonant optical readout. We need to integrate the two mirrors of each Fabry-Perot sensing cavity in two separate systems, which must be cold, massive and of high mechanical Q factor (below we give a less generic qualification), otherwise the thermal noise would be unacceptably large. We are thus led to the concept of a GW detector based on a massive *dual sphere* system of resonators: a hollow sphere which encloses a smaller solid sphere, see Fig. 1. Motion sensors in this system will be optical Fabry-Perot cavities formed by mirrors coated face to face to the inner surface of the hollow sphere and to the solid sphere, in either a PHC [14] or a TIGA [10] layout.

The main sources of noise are: i) thermal noise in the large detector masses, ii) back-action noise introduced by the radiation pressure, and iii) photon counting noise. Given the optical figures, the evaluation of all three contributions for our design is straightforward, as the spectrum of the resonant frequencies of the two spheres is known [12,13] once their material(s) and dimensions are fixed.

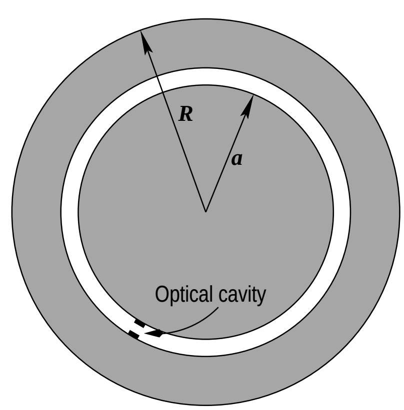

FIG. 1. A dual sphere GW detector with Fabry-Perot cavities as motion sensors.

Assuming the same material is used for both spheres, and that the inner one of radius a fills up almost completely the interior of the other (external radius R, internal radius $\gtrsim a$), the first quadrupole resonance of the outer hollow sphere is at the lowest frequency, while that of the inner solid sphere is 2-3 times higher. The frequency region in between is of particular interest: the GW signal drives the hollow sphere above resonance and the solid sphere below resonance. The responses of the two resonators are then out of phase by π radians and therefore the differential motion, read by the optical sensors, results in a signal enhancement; in this region only a small number of non-quadrupole resonances occur, which are not GW active. The pattern repeats for the two second quadrupole modes at higher frequency and so on. At a few specific frequencies above the first quadrupole resonance of the solid sphere, under the combined effect of the response to GW of all the quadrupole modes, their responses subtract and the sensitivity is reduced and eventually lost in a few narrow bands. In this higher frequency region, in addition to such loss of response, several resonances from the GW-inactive modes appear. While the spectral sensitivity would be still of some interest, we prefer for brevity to not discuss it here.

Let us assume that we only sense *radial* displacements and that the spherical symmetry of the resonators is not broken; using the notation of references [12] and [13] the response to a GW of the solid sphere at its surface and of the hollow at its *inner* surface (i.e. at the radius a) are respectively given by expressions of the type

$$u(\omega) = -\frac{1}{2}\sum_{n=0}^{\infty} b_n A_{n2}(a)\,\omega^2 \tilde{h}(\omega)\,L_{n2}(\omega)\ , \qquad (2)$$

where $A_{n2}(a)$ are radial function coefficients, b_n are the coefficients in the orthogonal expansion of the response function of each sphere, $L_{n2}(\omega)$ is the Lorentzian curve associated to the mode $\{n2\}$, the n-th quadrupole harmonic, and $\tilde{h}(\omega) \equiv \tilde{h}_{ij}(\omega)\, n_i n_j$ is the Fourier amplitude of the GW strain at the sensing point direction, defined by the unit radial vector $\mathbf{n}$ relative to the system's center of mass. Of course all these quantities must be calculated for either sphere.

Each sensor output is affected by thermal and back-action displacement noise spectral densities, which must be formed for both spheres:

$$S_{uu}^{[th+ba]}(\omega) = \sum_{nl} \frac{2l+1}{4\pi M}\, |A_{nl}(a)|^2\, |L_{nl}(\omega)|^2 \left[\frac{2kT\omega_{nl}}{Q_{nl}} + \frac{2l+1}{4\pi M}\, |A_{nl}(a)|^2 \sum_j |\mathcal{P}_l(\mathbf{n}\cdot\mathbf{n}_j)|^2\, S_{FF}^{ba} \right], \quad (3)$$

where k is Boltzmann's constant, T the sphere's thermodynamic temperature, and M the sphere's mass, whether solid or hollow. S_{FF}^{ba} is the back action force spectral density, $\mathcal{P}_l$ a Legendre polynomial, and $\mathbf{n}_j$ the spherical coordinates of the optical cavities ($\mathbf{n} \equiv \mathbf{n}_1$). The sum over j accounts for the fact that each sensor is additionally affected by the back-action noise forces exerted by the others. The back-action noise force is given by

$$S_{FF}^{ba}(\omega) = \frac{4}{c^2}\,(1-\zeta)^2\, h\nu_l\, F^2\, P_{in} \left[1 + \left(\frac{2FL_c\omega}{\pi c} \right)^2 \right]^{-1}, \quad (4)$$

where ν_l is the light frequency, c the speed of light, P_{in} the light power entering the cavity and ζ^2 the fraction of light reflected by the cavity at its resonance.

Assuming the noise in the spheres is uncorrelated [20], the total displacement spectral density is the sum of expressions like Eq. (3) for each sphere, plus a photodetector noise term, the *shot noise*. The total *strain* noise spectral density is thus given by

$$S_{hh}(\omega) = \frac{S_{uu,\mathrm{hollow}}^{[th+ba]}(\omega) + S_{uu,\mathrm{solid}}^{[th+ba]}(\omega) + S_{uu}^{shot}(\omega)}{|u_{\mathrm{hollow}}(\omega) - u_{\mathrm{solid}}(\omega)|^2/|\tilde{h}(\omega)|^2}\ . \quad (5)$$

Here $S_{uu}^{shot}(\omega)$ is the photodetector noise which can be written as

$$S_{uu}^{shot}(\omega) = 4\cdot 10^{-33} \left[1 + \left(\frac{2FL_c\omega}{\pi c} \right)^2 \right] \frac{1}{F^2 P}\ \mathrm{m^2/Hz}\ . \quad (6)$$

We may now consider the actual GW sensitivity of a system like this. We take as reference an operation at the Standard Quantum Limit. The SQL is reached at laser powers such that the shot noise and the radiation pressure equally contribute to the total noise and, at the same time, the back-action noise overcomes the mechanical resonator thermal noise. Note that, since the cavity is short, order of 1 *cm*, the finesse can be made very high, order of 10^6 and beyond, before loosing signal strength, and thus the SQL can be approached at laser powers of the order of 8 W. Moreover, since the bandwidth Δf is expected to be wide, $\Delta f \simeq f$, the SQL condition for the mechanical resonators $kT/Q = h\Delta f/4$ allows $Q/T = 10^8\,\mathrm{K}^{-1}$, not an impossible figure for materials such as sapphire, which shows $Q > 10^8$ at $T < 10\,\mathrm{K}$. Given that one is able to approach the SQL in the readout, then one needs a large cross-section to GWs. The resonance frequency fixed, the latter scales as ρv_s^5, where ρ is the density of the material and v_s the velocity of sound. Again sapphire shows up as a good choice, as it has one of the largest sound velocities, $v_s = 10\,\mathrm{km/s}$ and a density $\rho = 4\cdot 10^3\,\mathrm{kg/m^3}$.

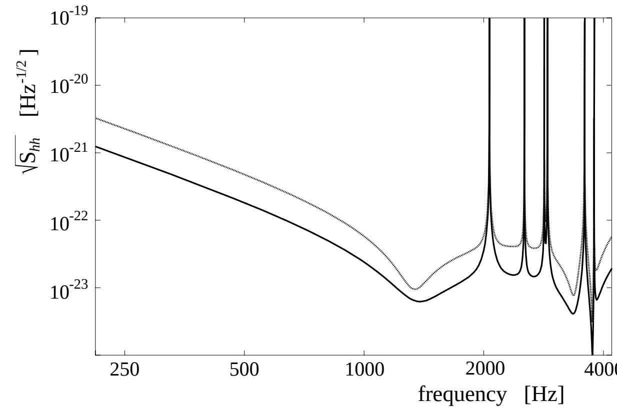

FIG. 2. Sensitivity of a dual sphere GW detector equipped with 10^6 finesse, 2 cm long Fabry-Perot cavities as motion sensors. The resonators' material is sapphire: the fundamental mode frequency of the outer (hollow) sphere is 1300 Hz while that of the solid (inner) sphere is 3370 Hz. The system has a total mass of about 30 + 8 tons, and an outer radius $R = 1.3\,\mathrm{m}$. Black curve: sensitivity corresponding to imposing the SQL at 1.5 kHz ($P = 8\,\mathrm{W}$, $Q/T = 5\cdot 10^8\,\mathrm{K}^{-1}$). Gray curve: sensitivity corresponding to $P = 1\,\mathrm{W}$, $Q/T = 10^8\,\mathrm{K}^{-1}$. Above 2 kHz the sensitivity is contaminated by spurious resonances of the spheres' non-quadrupole modes.

For the purpose of discussing the concept of our proposed configuration, we thus take sapphire as the material for *both* spheres. It may not be impossible to construct a large system with this material, for instance by silicate bonding of smaller pieces, as such a bonding procedure preserves the mechanical Q [21]. Sapphire, acting as substrate of the mirrors, will also minimize thermoelastic effects at low temperatures [22].

A sapphire detector with $R = 1.3\,\mathrm{m}$ and $a = 0.8\,\mathrm{m}$, with a small gap in between to place the motion sensing optical cavities, would give an interestingly low strain spectral noise in a rather wide frequency band in the kHz region, see Fig. 2. Here we plot the SQL spectral strain noise, when the radiation pressure noise is matched to the shot noise at 1.5 kHz. The spectral strain noise is also shown for a lower light power $P = 1\,\mathrm{W}$, possibly more amenable to cryogenic operation at $T \simeq 1\,\mathrm{K}$, but still giving an in-

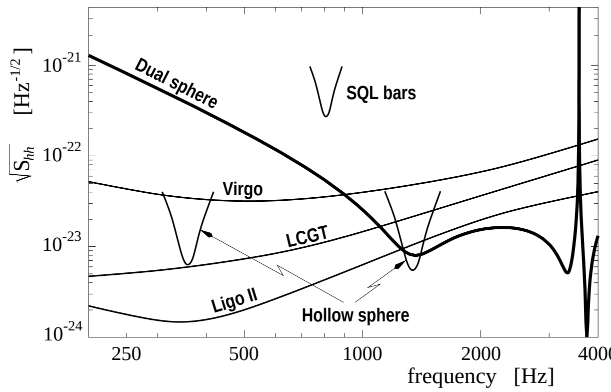

FIG. 3. Spectral strain sensitivities of various GW detectors, together with that of the "dual sphere" (see text).

teresting performance. Note that the spectral sensitivity is contaminated by the thermal noise peaks of the non-quadrupole resonances [12,13]. The problem of unwanted narrow resonances in the sensitive frequency band is also present in the case of interferometric detectors and methods have been devised to filter them out [23,24].

Figure 3 shows a comparative plot of the sensitivities of various GW detectors to come: initial VIRGO [23], the cryogenic interferometer LCGT [25], the advanced LIGO II design [26], the currently operating bars if pushed to their SQL, and our proposed dual sphere with the non-quadrupole resonances shown in Fig. 2 suppressed for clarity. The large drop in sensitivity of the latter, indicated by the prominent spike towards the right end of the figure is due to signal cancellation at a frequency $\omega_\star$ for which $u_{\rm hollow}(\omega_\star) = u_{\rm solid}(\omega_\star)$, which causes the denominator in eq. (5) to vanish. The presence of such a frequency $\omega_\star$ is expected on the basis of the intrinsic structure of eq. (5), and therefore it must be taken into consideration when one chooses materials and dimensions. As can be seen, the dual sphere system favorably compares with the best foreseen GW detectors, especially in the high frequency region where e.g. relatively small mass ($10M_\odot$) BH-BH mergers are expected [27].

In the end, the system we propose may still look like a two-mode system in that the most useful band is obtained between the two first quadrupole resonances of the two spheres. However the concept we propose allows one to choose such two frequencies with a lot of freedom, and in fact to open considerably the band with respect to systems which make use of resonant secondary masses to get the two-mode operation.

We occasionally made reference to practical issues for the realization of this system. This was not intended to assess its actual feasibility, but only to mention that a few relevant details do not lead to requests impossible to satisfy. It remains to make a careful analysis of the feasibility, especially in respect to construction, suspensions, cooling, etc., for such a massive system; we feel that so much experience continues to be acquired in this field, that the outcome may well be in the positive direction.

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