Thermal noise in a high Q cryogenic resonator

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Abstract

In order to evaluate the feasibility of a mixed mechanical and electrical multimode matching network for a resonant gravitational wave detector, current noise measurements were performed on a high quality factor LC resonator, based on a superconducting coil, by using a dc SQUID weakly coupled to the coil. We present a method to identify different noise sources in the system by their dependence on the temperature or on the resonator quality factor. Measurements performed at quality factors up to 10^6 in the temperature range 1.2 - 4.2 K, show that the total measured noise is only contributed by the resonator thermal noise.

1 Introduction

The present resonant gravitational wave detectors [1] employ multimode networks for broadband impedance matching between the main mechanical resonator and sensing element (usually a 2 tons aluminum cylindrical bar) and the signal amplifier (usually a dc SQUID).

In practice, those networks are constituted by one or two mechanical resonators connected to the bar (for a lumped model of a multimode detector see ref. [2]), with stepped down values of the masses; to benefit from higher mechanical amplification, the resonance frequencies of the network resonators, when uncoupled, are tuned to that of the bar. An optimal matching network permits to approach the detector sensitivity limit, determined by the noise temperature of the amplifier and attained when the detector is lossless [3], even if its modes have relatively low quality factors [4]. It moreover provides a larger bandwidth than the one mode systems.

It has been shown that the most of the improvement to be gained by using a multimode system instead of a single mode is obtained with three-four mode system [5]. For this reason some three mechanical mode systems (the main resonator plus a two-mode transducer) have been tested [6] but no one of these is employed in the cryogenic detectors now operating.

The AURIGA detector [1] and all the detectors with resonant capacitive transducer are systems (Fig. 1) with two mechanical modes (bar and transducer modes), plus an electrical mode which is kept weakly coupled to the mechanical ones (that is at frequencies far enough from those of the mechanical modes) because its losses would reduce the very high mechanical quality factors and hence the detector sensitivity. In this case the detector is equivalent to a

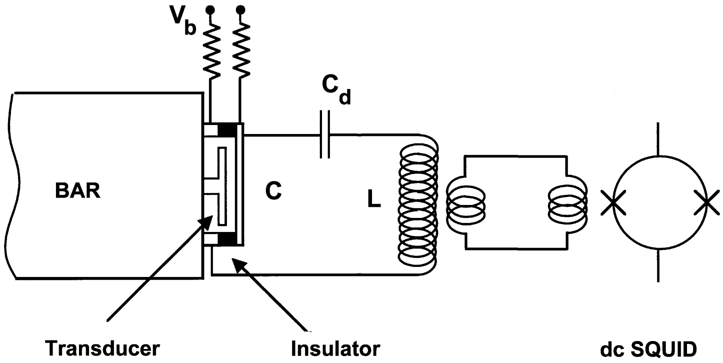

Figure 1: *The bar vibrations are converted into electrical signal by a capacitive mushroom transducer whose lowest symmetrical flexural mode is tuned on the first longitudinal mode of the bar. Bar and transducer form a system of two coupled mechanical oscillators; the third electrical oscillator is composed by the transducer capacitance C and by the matching transformer primary inductance L. The decoupling capacitance $C_d \gg C$ permits to charge the transducer with the bias voltage V_b.*

two mode system because the electrical mode is not tuned on the mechanical ones. Numerical analyses show [7] that the tuning of the electrical mode on the mechanical ones can improve the detector performance, particularly as regards the bandwidth, only if two requirements are fulfilled: the quality factor of the electrical mode must be of the same order of magnitude of those of the mechanical modes, that is 10^6, and the noise due to the electrical mode must correspond to the fundamental limiting noise of the passive components, that is the thermal noise.

Electrical resonators based on a low loss superconducting coil whose inductance is similar to that of the primary coil in the Auriga matching transformer and operating with quality factor of the order of 10^6, have been already realized [8, 9]. The further improvements in the reduction of the coil losses entitle us to believe that a quality factor of electrical mode of 10^6 can be achieved. As regards the requirement of thermal noise for the electrical mode, it is easy to guess a few mechanisms which can lead to an excess noise. For example, the vibrations of the coil with respect to the magnetic field which is trapped in the coil superconducting housing, or the ambient magnetic field fluctuations which penetrate the coil superconducting housing, can give rise to an excess gaussian noise. Flux jumps in the superconducting coil or in its superconducting housing are a possible source of non gaussian noise. This can be easily distinguished from the thermal noise but is still a problem for the gravitational wave detector because it mimics the gravitational bursts. To evaluate the practical feasibility of the mixed mechanical-electrical matching network, we have measured the thermal noise of a high Q electrical resonator with a dc SQUID as is shown in Fig. 2. We also present a method for the noise characterization based on measurements performed at different values of the resonator quality factor and temperatures. In fact, as it will be shown in the next sections, different noise sources can be identified by their temperature or Q dependence.

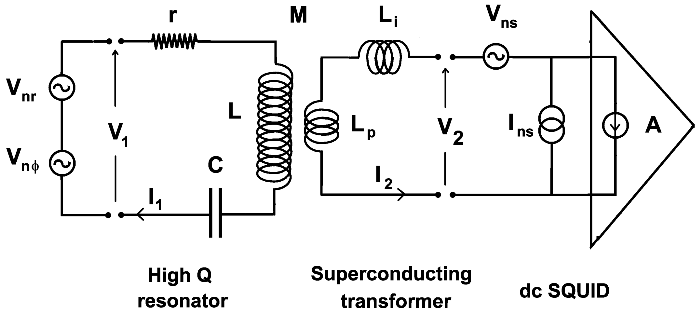

Figure 2: *Schematic of the resonator with the SQUID used to measure the noise currents.*

2 A model for the resonator current noise

In this section we show a model to describe the noise currents circulating in a LC electrical resonator coupled to a dc SQUID as in Fig. 2. As we consider small values of the coupling, we can neglect in our calculation the effects of the SQUID input dynamic impedance [10, 11] on the resonator. The resonator energy dissipation is modelled by the resistance r, which acts as a Nyquist voltage noise source V_{nr}. A second voltage noise source $V_{n\phi}$ accounts for any residual magnetic flux noise picked up by the coil of inductance L. The corresponding power spectra are $S_{vr} = 2k_B Tr$ and $S_{v\phi}$, where k_B is the Boltzmann constant and T the resonator temperature in Kelvin.

The magnetic flux changes through the coil L are transferred to the SQUID by coupling, with mutual inductance M, a superconducting pick-up of inductance L_p. This pick-up and the dc SQUID input coil L_i form the superconducting transformer. The SQUID is modelled by an ideal current amplifier with gain A, a current noise generator I_{ns} in parallel to the input port and a voltage noise generator V_{ns} in series to it. The corresponding power spectra are S_{is} and S_{vs}. By solving the equations:

$$\begin{aligned} I_1(r + i\omega L - \frac{i}{\omega C}) + I_2 i\omega M &= V_1 \\ I_1 i\omega M + I_2(i\omega L_p + i\omega L_i) &= V_2 \ , \end{aligned} \tag{1}$$

in the frequency domain, the current I_1 and I_2 flowing in the circuit are calculated as:

$$\begin{aligned} I_1 &= f_{11}(\omega)V_1 + f_{12}(\omega)V_2 \\ I_2 &= f_{21}(\omega)V_1 + f_{22}(\omega)V_2 \ , \end{aligned} \tag{2}$$

where $f_{ij}(\omega)$ are the transfer functions of the circuit. If the noise sources are not correlated, the power spectra of the noise current produced by the noise sources in the resonator and in the transformer are:

$$\begin{aligned} S_{I1} &= |f_{11}(\omega)|^2 \big(S_{vr} + S_{v\phi}\big) + |f_{12}(\omega)|^2 S_{vs} \\ S_{I2} &= |f_{21}(\omega)|^2 \big(S_{vr} + S_{v\phi}\big) + |f_{22}(\omega)|^2 S_{vs} \end{aligned} \tag{3}$$

Each power spectrum $S_{I1,I2}$ gives the variances $\sigma^2_{I1,I2}$ of the noise currents I_{n1} and I_{n2}, that is the expected values of the square of the noise currents, as their mean values are null:

$$E(I^2_{n1,n2}) = \sigma_{I1,I2} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{I1,I2}(\omega)d\omega \qquad (4)$$

The transfer function $f_{11}(\omega)$ gives the current I_1 generated by an applied voltage $V_1(\omega)$ with $V_2 = 0$:

$$|f_{11}(\omega)|^2 = \frac{\omega^2/L^2_{eq}}{\frac{\omega^2\omega_0^2}{Q^2} + (\omega^2-\omega_0^2)^2} \qquad (5)$$

This is the transfer function of the resonator coupled with the transformer: its behaviour is the same of a standard rLC resonator with resonance $\omega_0 = \frac{1}{\sqrt{L_{eq}C}}$ and quality factor $Q = \frac{\omega_0 L_{eq}}{r}$. The inductance is $L_{eq} = L - \frac{M^2}{L_p+L_i}$, where the term $\frac{M^2}{L_p+L_i}$ is the component reflected from the transformer. The quality factor Q depends on the free decay time constant $\tau = 2Q/\omega_0$, as it could be easily obtained by solving the equations (1) in the time domain. We remark that by Eqs. 3 and 5, the mean value of the square the noise current in the resonator due only to the thermal noise $S_{vr} = 2k_BTr$ is:

$$E(I^2_{n1r}) = 2k_BTr\frac{1}{L^2_{eq}}\frac{Q}{2\omega_0} = \frac{k_BT}{L_{eq}} \ , \qquad (6)$$

(where we used $\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\omega^2/(\frac{\omega^2\omega_0^2}{Q^2}+(\omega^2-\omega_0^2)^2)\right)d\omega = \frac{Q}{2\omega_0}$) in agreement with the energy equipartition theorem for the resonator:

$$\frac{1}{2}L_{eq}E(I^2_{n1r}) = \frac{1}{2}k_bT \ . \qquad (7)$$

The other transfer functions are

$$|f_{21}(\omega)|^2 = |f_{12}(\omega)|^2 = \frac{\frac{M^2\omega^2}{(L_p+L_i)^2L^2_{eq}}}{\frac{\omega^2\omega_0^2}{Q^2} + (\omega^2-\omega_0^2)^2} \qquad (8)$$

$$|f_{22}(\omega)|^2 = \frac{L^2}{(\omega(L_p+L_i)L_{eq})^2}\frac{\frac{\omega^2\omega_1^2}{Q_1^2} + (\omega^2-\omega_1^2)^2}{\frac{\omega^2\omega_0^2}{Q^2} + (\omega^2-\omega_0^2)^2} \ , \qquad (9)$$

where $\omega_1 = 1/\sqrt{LC}$ and $Q_1 = \omega_1 L/r$. The power spectrum of the total current noise at the SQUID amplifier input is obtained by adding to S_{I2} the contribution S_{is} due to the noise generator I_{ns}.

$$S_{IS} = |f_{21}(\omega)|^2S_{vr} + |f_{21}(\omega)|^2S_{v\phi} + |f_{22}(\omega)|^2S_{vs} + S_{is} \qquad (10)$$

3 Experimental apparatus

The LC resonator and SQUID are housed in a superconducting shielded box cooled to liquid helium temperatures (Fig. 3). All the parts of this box are electroplated with a 50 μm thick Pb40%Sn coating. This superconducting coating

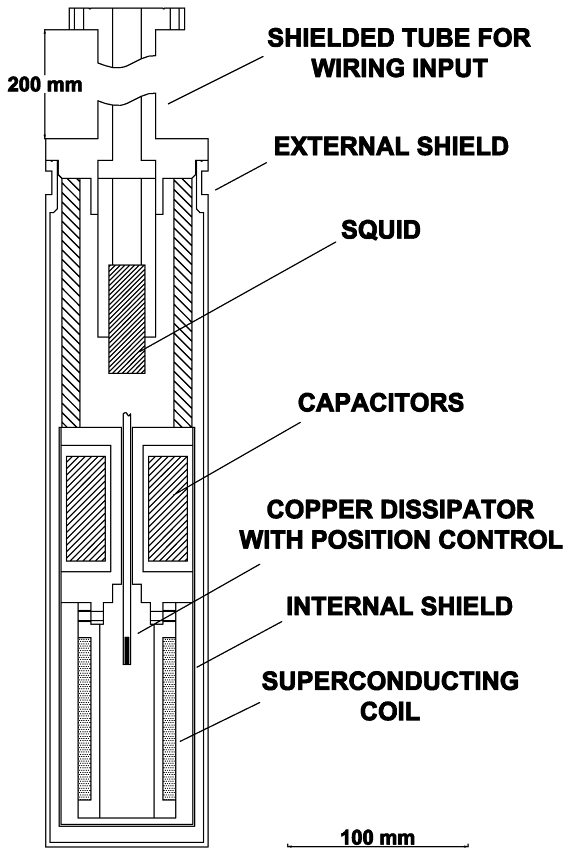

Figure 3: *Schematic of the apparatus.*

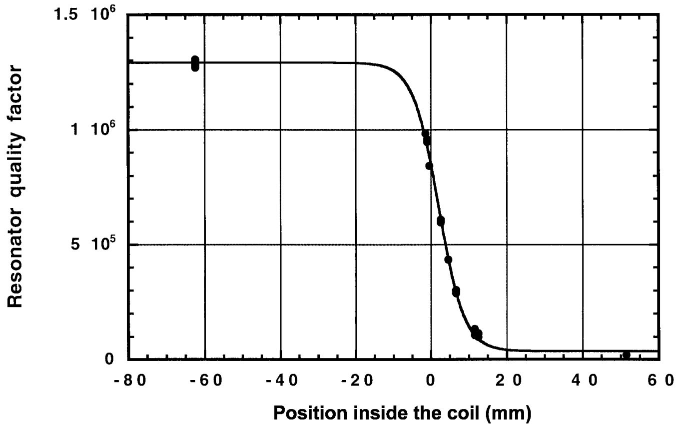

Figure 4: *Resonator quality factor as a function of the position of the copper cylinder end inside the coil.*

has the function to shield the resonator and the SQUID from the ambient magnetic field. Moreover the coating avoids the eddy currents dissipation that could reduce the resonator quality factor. The lead alloy with 40% of tin has been employed for its properties of hard superconductor [12], in order to minimize the losses due to fluxons motion. The outer shield is made in two pieces, vacuum-tight assembled with a PbSn o-ring. The shield has no tapered holes and the o-ring is pressed by two external clamps. The main wiring port is made by a tube with a length/diameter ratio of about 12.5, to reduce external magnetic field input. The LC resonator is enclosed in a second internal shield with 73 mm of internal diameter, where capacitor and superconducting coil are separated by a shield. The wiring ports in these shields are also small holes with a length/diameter ratio greater than 10.

The low dissipation coil was made by winding a formvar insulated 75 μm diameter NbTi wire (100 μm total diameter) on to a cylindrical teflon coil-holder. The coil holder is 110 mm in height and has an internal hole of 32 mm diameter. The total number of wire turns is 7042 in 17 layers. The coil has an internal diameter of 40 mm, external diameter of 54 mm and is 90 mm in height. The value of the inductance was measured at 4.2 K as $L = (0.554 \pm 0.005)\ H$ inside the superconducting shield. A commercial teflon capacitor [14] with a room temperature capacity of $\simeq 50\ nF$ was connected to the coil to build the cold resonator. With this capacity the oscillation frequency was measured as $\nu = 1/(2\pi\sqrt{LC}) = 926.78\ Hz$, so that the cold value of the capacitor was $C = (53.2 \pm .5)\ nF$.

In the coil holder is made an axial hole (Fig. 3) to permit the insertion of a dissipator which is used to change the resonator quality factor. The dissipator consists of a copper cylinder with height 15 mm and diameter 2 mm. The 4.2 K electrical conductivity of this copper is $\sigma_{cu} \sim 5.25 \times 10^9\ (\Omega m)^{-1}$. By applying a boundary elements numerical method [13] it can be estimated that the power dissipated by eddy current losses should reduce the quality factor to $\sim 10^4$ when the copper cylinder is fully inserted into the superconducting coil. A motion feedthrough allows to move the cylinder from the inside of the superconducting support of the capacitors (where it is shielded from the coil field and does not add dissipation) into the middle of the coil. In Fig. 4 the resonator quality factor is shown as a function of the position of the copper cylinder in the coil. At the lower quality factor the dissipation is mainly due to the copper cylinder, whereas the upper limit to Q is mainly due to dielectric dissipations of the teflon capacitors [8].

We used a commercial Quantum Design dc SQUID operated in closed loop. The SQUID gain as current amplifier is $A = M_s G = \ (3.56 \pm 0.03) \times 10^6 \ V/A$, where $M_s \simeq 1.10 \times 10^{-8} \ H$ is the mutual inductance between SQUID loop and input coil L_i, and $G = (3.22 \pm 0.03) \times 10^{14} \ V/Weber$ is its sensitivity to magnetic flux changes. The pick-up coil is made by a single turn of 75 μm superconducting wire wound around a 4 mm diameter PVC holder. Its calculated inductance is $\sim 1.1 \times 10^{-8} \ H$.

The wiring from room temperature are thermalized on a copper holder that is in good thermal contact with the top shield flange. In the cool-down operations some He gas ($\sim$ 100 mB stp) is introduced into the box, and is then pumped out when the system is at 4.2 K. The copper holder temperature is monitored by a germanium thermometer. The apparatus is cooled down by inserting it slowly in a 60 l helium dewar equipped with a μ-metal shield. Temperatures lower than 4.2 K are obtained by pumping the liquid helium bath. The whole system stays over a platform which is suspended by active air springs and a system of bellows is used to reduce the vibrations coming from the 50 mm diameter pumping line. The temperature stabilization was obtained with different methods depending on the range. At 4.2 K we did not used any stabilization system, as the temperature fluctuations induced by the atmospheric pressure changes are negligible for our sake. At 3 K we stabilized the bath pressure and hence the temperature at better than .1 mB with a feedback loop acting on the effective pumping speed. At temperature lowers than 2.17 K the bath temperature can be directly regulated by the feedback loop with a stability of 10 mK [15].

4 Noise measurement method

To measure the quality factor Q and resonance frequency $f_0 = \omega_0/2\pi$ of the resonator the LC resonator is excited [16] and then let free to decay. The signal from the SQUID is sent to a lock-in amplifier with reference ω_{lk} and the measured magnitude and phase are sampled. The phase velocity is proportional to $\omega_{lk} - \omega_0$ and ω_0 is evaluated with a linear fit of the sampled phase versus time. The quality factor is $Q = \pi \nu_0 \tau$ where τ is the time constant evaluated from an exponential fit of the sampled magnitude versus time. The decay time was also measured with the SQUID switched off to evaluate the effect of the SQUID dynamic impedance: the oscillation amplitude is measured at $t = 0 \ s$, the SQUID is switched off and on again at time t_1. A negligible effect of the SQUID on the resonator was found at all the working quality factors and temperatures.

To measure the unperturbed system noise, the SQUID output is sent to a lock-in amplifier with reference ω_{lk} and time constant τ_{lk}. The two phases X_n and Y_n are sampled together at regular time intervals. The voltage power spectrum at SQUID output is AS_{IS}, where S_{IS} was calculated in Eq. 10. It

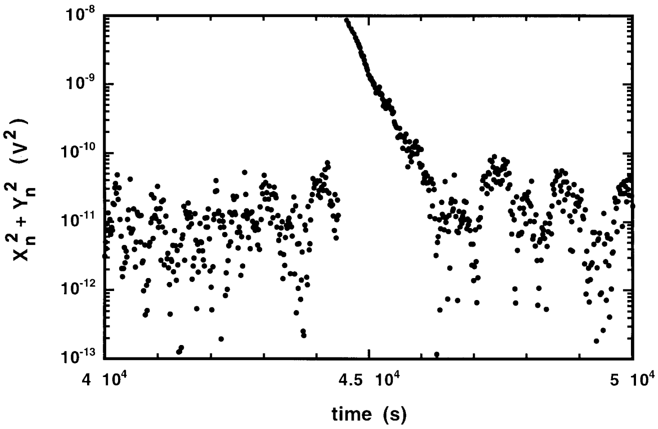

Figure 5: *SQUID output squared amplitude $R_n^2 = X_n^2 + Y_n^2$ as a function of time at $Q \sim 10^6$. The phases from the lock-in with $\tau_{lk} = 3$ s were sampled at 16 s, resonator time constant is $\tau \sim 446$ s. Umperturbed behaviour is shown together with an infrequent impulsive event, with energy more then 100 times the average one; after the sharp rising the system decays with its time constant.*

can be shown that, if:

$$\begin{aligned} |\omega_{lk} - \omega_0| &\ll \frac{1}{\tau_{lk}} \\ \tau &\gg \tau_{lk} \ , \end{aligned} \qquad (11)$$

the variance σ_n^2 of the statistical distribution of the phases X_n and Y_n is obtained by adding up these three factors:

$$\begin{aligned} \sigma_{ns}^2 &= \frac{A^2 S_{is}}{2\tau_{lk}} \\ \sigma_{nT}^2 &= \frac{A^2 M^2}{(L_p + L_i)^2}\Big(\frac{k_B T}{2L_{eq}}\Big) \\ \sigma_{nQ}^2 &= Q\frac{A^2 M^2}{(L_p + L_i)^2}\Big(\frac{S_{v\phi}(\omega_0)}{4\omega_0 L_{eq}^2} + \frac{S_{vs}(\omega_0)}{4\omega_0}\frac{M^2}{(L_p + L_i)^2 L_{eq}^2}\Big) \end{aligned} \qquad (12)$$

From a measure of σ_n^2 the thermal noise contribution σ_{nT}^2 can be obtained if the SQUID broadband noise contribution σ_{ns}^2 and the flux noise contribution and the SQUID voltage noise contribution σ_{nQ}^2, proportional to the resonator quality factor, are independently measured. We note that the thermal contribution and the flux noise contribution (term proportional to $S_{v\phi}(\omega_0)$) have the same dependence on the mutual inductance M, so that their relative magnitude is independent on the coupling if the resonant frequency is kept constant.

In Fig. 5 a selected SQUID output amplitude $R_n^2 = X_n^2 + Y_n^2$ time behaviour is shown. As expected the time scale of amplitude changes is comparable with the resonator time constant, except for a sharp rising characterising an infrequent impulsive event, that happen at a frequency of no more than 1/day. From each data set we eliminate any impulsive event with amplitude higher than 50 times the average one. In order to carefully recognize the sharp rising in the

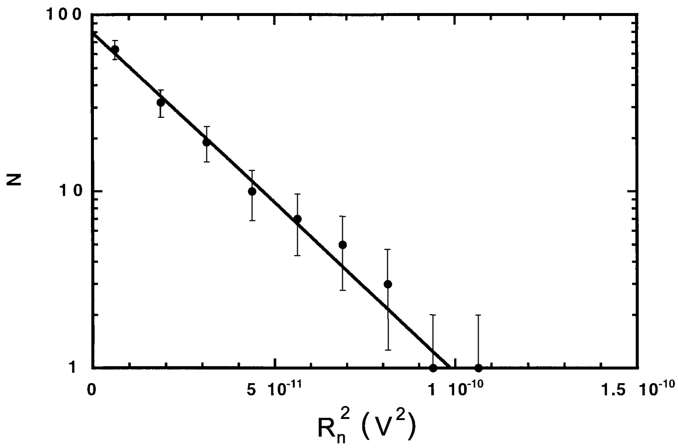

Figure 6: *Histogram of the number of samples of R_n^2 for each interval $\Delta_{R_n^2} = 1.2 \times 10^{-12}\ V^2$. The solid line fits the distribution Eq. 13 with σ_n^2 as fitting parameter.*

SQUID output, the lock-in phases sampling is performed faster than the resonator time constant, every 4 s or 16 s (depending on τ_{lk}). The collected data are then decimated to one every 2τ to obtain statistically uncorrelated data set. As the phases distributions are gaussian with the same variance σ_n^2 calculated in Eq. 12, the squared amplitude $R_n^2 = X_n^2 + Y_n^2$ is Boltzmann distributed:

$$F(R_n^2) = \frac{e^{-\frac{R_n^2}{2\sigma_n^2}}}{2\sigma_n^2} \tag{13}$$

To evaluate σ_n^2 we first check the Gaussian behaviour of each phase, then calculate $R_n^2 = X_n^2 + Y_n^2$. A weighted exponential fit of the histogram of R_n^2 with Eq. 13 gives then the variance σ_n^2 (Fig. 6). This method is less sensitive to any infrequent non-gaussian events than the direct calculation of the variance of the phases X_n^2 and Y_n^2.

The SQUID broadband noise contribution σ_{ns}^2 at ω_0 was estimated at each operating temperature by two different methods finding agreement. The power spectrum was first measured by a spectrum analyzer in a frequency band of some 100 Hz around ω_0. As it has been found to be white in this range, the contribution σ_{ns}^2 was estimated by sampling the SQUID output at a reference frequency of 10 Hz away from ω_0.

The current-to-voltage gain $K_m = \frac{AM}{L_i+L_p} = (7.52 \pm .03) \times 10^5\ \frac{V}{A}$ has been evaluated in a calibration run in which the response of the SQUID to a known current injected in the coil has been measured. As $L_p + L_i \simeq L_i \simeq 1.88\ \mu H$, from this calibration we also have an estimate of mutual inductance $M = (4.0 \pm 0.1) \times 10^{-7}\ H$, and of the coupling $k = \frac{M}{\sqrt{LL_p}} \sim 5 \times 10^{-3}$. These values are low enough to make negligible the SQUID input dynamic impedance and allow the application of the model of Sec. 2.

Measurements at 4 different system temperatures were performed. For each temperature lock-in output data set at different Q values are sampled. For each quality factor τ_{lk} and ω_{lk} values where chosen in agreement with Eqs. 11.

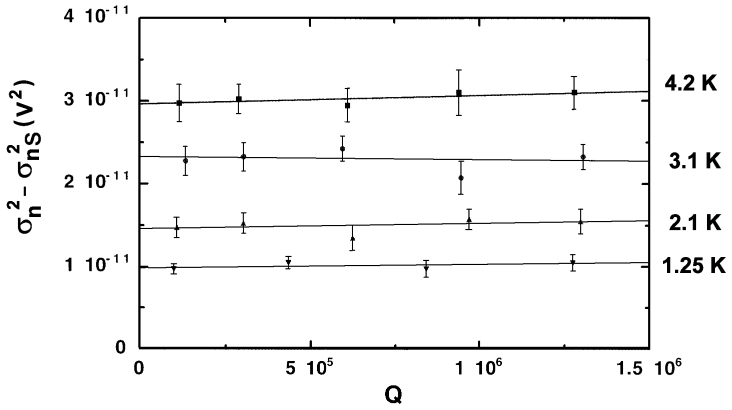

Figure 7: *Variance of the SQUID output noise less the wide band SQUID noise contribution at different temperatures as a function of the quality factor Q.*

T (K)	σ_{is}^2/τ_{lk} (V^2/s)	σ_{nT}^2 (V^2)	σ_{nQ}^2/Q (V^2)
1.25 ± 0.01	$(.75 \pm 0.07) \times 10^{-12}$	$(0.98 \pm 0.06) \times 10^{-11}$	$(4 \pm 9) \times 10^{-19}$
2.10 ± 0.01	$(1.0 \pm 0.1) \times 10^{-12}$	$(1.5 \pm 0.1) \times 10^{-11}$	$(0._5 \pm 1) \times 10^{-18}$
3.12 ± 0.01	$(1.1 \pm 0.1) \times 10^{-12}$	$(2.3 \pm 0.1) \times 10^{-11}$	$(-0._5 \pm 2) \times 10^{-18}$
4.19 ± 0.01	$(1.4 \pm 0.2) \times 10^{-12}$	$(3.0 \pm 0.2) \times 10^{-11}$	$(1 \pm 2) \times 10^{-18}$

Table 1: *Noise contributions data at different temperatures.*

5 Results and discussion

In Fig. 7 the values of $\sigma_n^2 - \sigma_{ns}^2$ as a function of the quality factor for each operation temperature are shown. The wide band noise contributions are given in Tab. 1 per unit lock-in time constant. As $\tau_{lk} \geq 1$ s, σ_{ns}^2 is always less than 10% of the total noise. For each temperature the thermal contribution σ_{nT}^2 and the contribution σ_{nQ}^2 proportional to the quality factor are evaluated by a linear fit and are also reported in Tab. 1. In Fig. 8 the thermal contribution is shown as a function of the liquid helium bath temperature. The theoretical expectation, $\sigma_{nT}^2 = K_m^2 T \frac{k_B}{2L_{eq}} = (7.1 \pm 0.1) \times 10^{-12}$ $\frac{V^2}{K}$ T, calculated by K_m and L_{eq} as measured in the calibration run, is also plotted.

We first point out that, as is shown in Fig. 7, there is not a noise contribution proportional to the quality factor within errors. As expected the back-action noise (term proportional to S_{vs} in eq. 12) is made negligible by our low coupling $k \sim 5 \times 10^{-3}$, but a very good shielding was needed to reduce the magnetic flux noise pick-up (term proportional to $S_{v\phi}$) of the coil well below the thermal noise. As an example, in Fig. 9 is shown the output of poorly shielded resonator, where a strong linear dependence on the quality factor is observed. From the data shown in Tab. 1, we can limit the noise spectral power density of the flux induced voltage noise as $S_{v\phi}(\omega_0) < 2 \times 10^{-26}$ V^2/Hz. This figure limits to $\sim 3 \times 10^{-34}$ Wb^2/Hz the spectral density of the flux noise picked-up by the coil at ω_0, corresponding to a fluctuating magnetic field of $\sim 10^{-18}$ $Tesla/\sqrt{Hz}$ into the coil shield.

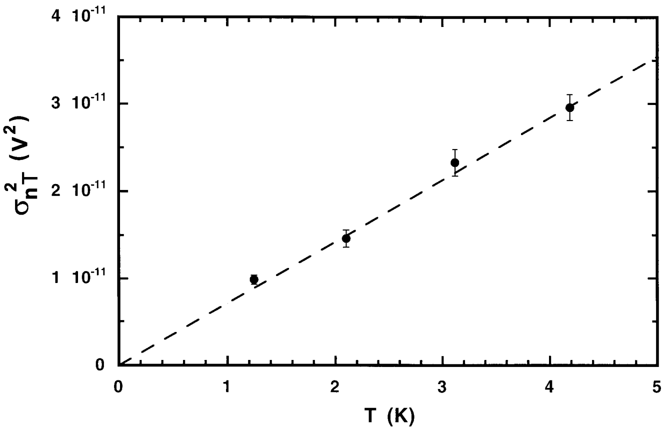

Figure 8: *Thermal contribution σ^2_{nT} as a function of the liquid helium bath temperature. The dashed line is the theoretical expectation of Eq. 12 with no free parameters.*

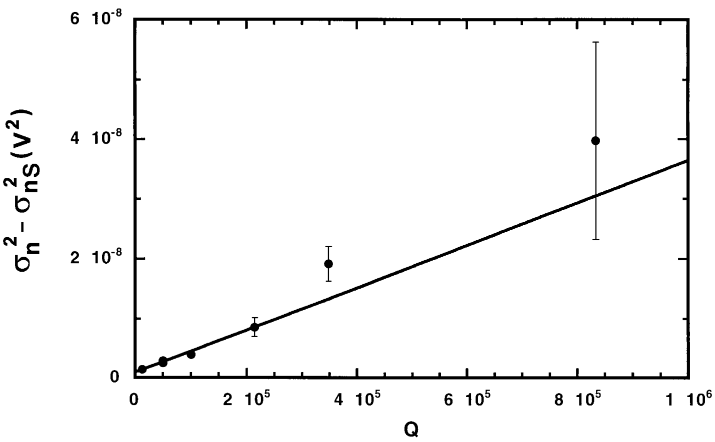

Figure 9: *Variance of the SQUID output noise less the wide band SQUID noise contribution as a function of the quality factor Q in a bad shielding condition.*

As a second point, the existence of a voltage noise source due to the dissipative element of the resonator and proportional to the temperature is confirmed by the data shown in Fig. 8, that are consistent with our theoretical model within the calibration errors. The temperature of the dissipative element, as evaluated from the noise measurements, is coincident with the bath temperature measured by the thermometer. In conclusion it is demonstrated that the resonator-SQUID system noise has a thermal behaviour in the measurement range.

These results support the feasibility of a mixed mechanical and electrical multimode matching network for a resonant gravitational wave detector. In fact the occurring of infrequent well-defined extra-noise events as shown in Fig. 5, which are not relevant in our noise measurements as discussed above, will not affect a gravitational wave detector performances too, as these events are easily rejected by the standard protocol of coincidence between different detectors. Moreover extra-noise events in the electrical resonator occur at a frequency much lower than the extra-noise events actually observed in a detector. The events we have detected, that happen at a frequency of no more than 1/day, could be generated by mechanical disturbances, by external electromagnetic disturbances on the SQUID electronics, or flux jumps in the coil, in the shield or in the SQUID. As the mechanical insulation of the LC resonator in the present experiment is a few orders of magnitude worse than that of a matching transformer mounted on the gravitational wave detector, the extra-noise events occurrency, if mechanically induced, should be dramatically decreased if our shielded coil should be installed in AURIGA as primary coil of the matching transformer.

We observe at last that by Eq. 12 the thermal noise and the interference noise are both proportional to the coupling. This means that $S_{v\phi}$ remains neglibile also when the coupling is increased as required for the sake of the matching network. As regards the SQUID voltage noise S_{vs}, we find on the basis of a theoretical estimate [10] that the back-action noise contribution of our SQUID should equalize the thermal noise contribution, at $Q = 10^6$ and coupling $k = 0.4$.

The knowledge of the voltage noise level of the dc SQUID employed in the AURIGA detector would be of fundamental importance for the achievement of the noise matching and, hence, of the detector sensitivity limit. To our knowledge this would be also the first back-action measurement of a low noise dc SQUID.

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